# The MAD Program (Methodical Accelerator Design) Version 8.13/8 <br> User's Reference Manual 

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#### Abstract

MAD is a tool for charged-particle optics in alternating-gradient accelerators and beam lines. It can handle very large and very small accelerators and solves various problems on such machines.

Features include: Linear lattice parameter calculation, linear lattice matching, transfer matrix matching, survey calculations, error definitions, closed orbit correction, particle tracking, chromatic effects and resonances, file services, subroutines, intra-beam scattering, Lie-algebraic analysis, debugging services. Most of the Lie-algebraic algorithms used have been developed at Maryland University by Alex Dragt and his collaborators.

The present revision of this manual is valid for Version 8.13 of MAD. It contains the following enhancements: Option to allow additive error definitions and corrector settings, option to save machine structure as a line definition, COMMENT command, improved synchrotron radiation (separate switches in tracking for damping and quantum effects), correct rendering of tunes for large momentum deviation, generation of track tables in selected observation points.


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## Preface

The MAD framework should make it easy to add new features in the form of program modules. The authors of MAD hope that such modules will also be contributed and documented by others. Future updates to this manual will be published in the form of complete chapters which either replace existing ones, or document new features. As required, the table of contents and the indices will be revised. The contributions of other authors are acknowledged in the relevant chapters.

Misprints and obscurity are almost inevitable in a manual of this size. Comments from readers are therefore most welcome. They may also be sent to one of the following BITNET addresses:

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## Chapter 1. Conventions

### 1.1 Reference System

The accelerator and/or beam line to be studied is described as a sequence of beam elements placed sequentially along a reference orbit. The reference orbit is the path of a charged particle having the central design momentum of the accelerator through idealised magnets with no fringe fields (see Figure 1.1).

The reference orbit consists of a series of straight line segments and circular arcs. It is defined under the assumption that all elements are perfectly aligned. The accompanying tripod of the reference orbit spans a local curvilinear right handed coordinate system ( $x, y, s$ ). The local $s$-axis is the tangent to the reference orbit. The two other axes are perpendicular to the reference orbit and are labelled $x$ (in the bend plane) and $y$ (perpendicular to the bend plane).

### 1.2 Closed Orbit

Due to various errors like misalignment errors, field errors, fringe fields etc., the closed orbit does not coincide with the reference orbit. It also changes with the momentum error. The closed orbit is described with respect to the reference orbit, using the local reference system ( $x, y, s$ ). It is evaluated including any nonlinear effects.

MAD also computes the betatron and synchrotron oscillations with respect to the closed orbit. Results are given in the local $(x, y, s)$-system defined by the reference orbit.

### 1.3 Global Reference System

The reference orbit of the accelerator is uniquely defined by the sequence of physical elements. The local reference system $(x, y, s)$ may thus be referred to a global Cartesian coordinate system $(X, Y, Z)$ (see Figure 1.2). The positions between beam elements are numbered $0, \ldots, i, \ldots, n$. The local reference system $\left(x_{i}, y_{i}, z_{i}\right)$ at position $i$, i.e. the displacement and direction of the reference orbit with respect to the system $(X, Y, Z)$ are defined by three displacements ( $X_{i}, Y_{i}, Z_{i}$ ) and three angles ( $\theta_{i}, \phi_{i}, \psi_{i}$ ) The above quantities are defined more precisely as follows:

Displacement of the local origin in $X$-direction.
Y Displacement of the local origin in $Y$-direction.
Z Displacement of the local origin in $Z$-direction.
[ THETA] Angle of rotation (azimuth) about the global $Y$-axis, between the global $Z$ axis and the projection of the reference orbit onto the ( $Z, X$ ) -plane. A positive angle $\theta$ forms a right-hand screw with the $Y$-axis.

PHI] Elevation angle, i.e. the angle between the reference orbit and its projection onto the ( $Z, X$ )-plane. A positive angle $\phi$ correspond to increasing $Y$. If only horizontal bends are present, the reference orbit remains in the ( $Z, X$ )-plane. In this case $\phi$ is always zero.
[
PSI] Roll angle about the local $s$-axis, i.e. the angle between the intersection of the $(x, y)$ and ( $Z, X$ )-planes and the local $x$-axis. A positive angle $\psi$ forms a right-hand screw with the $s$-axis.

The angles $(\theta, \phi, \psi)$ are not the Euler angles. The reference orbit starts at the origin and points by default in the direction of the positive $Z$-axis. The initial local axes ( $x, y, s$ ) coincide with the global axes ( $X, Y, Z$ ) in this order. The six quantities ( $X_{0}, Y_{0}, Z_{0}, \theta_{0}, \phi_{0}, \psi_{0}$ ) thus all have zero initial values by default. The program user may however specify different initial conditions.

Internally the displacement is described by a vector $V$, and the orientation by a unitary matrix $W$. The column vectors of $W$ are the unit vectors spanning the local coordinate axes in the order $(x, y, s) . V$ and $W$ have the values

$$
V=\left(\begin{array}{l}
X \\
Y \\
Z
\end{array}\right), \quad W=\Theta \Phi \Psi
$$

where

$$
\Theta=\left(\begin{array}{ccc}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{array}\right), \quad \Phi=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{array}\right), \quad \Psi=\left(\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right) .
$$

The reference orbit should be closed and it should not be twisted. This means that the displacement of the local reference system must be periodic with the revolution frequency of the accelerator, while the position angles must be periodic (modulo $2 \pi$ ) with the revolution frequency. If $\psi$ is not periodic (modulo $2 \pi$ ), coupling effects are introduced. When advancing through a beam element, MAD computes $V_{i}$ and $W_{i}$ by the recurrence relations

$$
V_{i}=W_{i-1} R_{i}+V_{i-1}, \quad W_{i}=W_{i-1} S_{i}
$$

The vector $R_{i}$ is the displacement and the matrix $S_{i}$ is the rotation of the local reference system at the exit of the element $i$ with respect to the entrance of the same element. The values of $R$ and $S$ are listed below for each physical element type.

### 1.3.1 Straight Elements

In straight elements the local reference system is simply translated along the local $s$-axis by the length of the element. This is true for

- Drift spaces,
- Quadrupoles,
- Sextupoles,
- Octupoles,
- Solenoids,
- RF cavities,
- Electrostatic separators,
- Closed orbit correctors,
- Beam monitors.

The corresponding $R$ and $S$ are

$$
R=\left(\begin{array}{l}
0 \\
0 \\
L
\end{array}\right), \quad S=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)
$$

A rotation of the element about the $s$-axis has no effect on $R$ and $S$, since the rotations of the reference system before and after the element cancel.

### 1.3.2 Bending Magnets

Bending magnets have a curved reference orbit. For both rectangular and sector bending magnets

$$
R=\left(\begin{array}{c}
\rho(\cos \alpha-1) \\
0 \\
\rho \sin \alpha
\end{array}\right), \quad S=\left(\begin{array}{ccc}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{array}\right)
$$

where $\alpha$ is the bend angle. A positive bend angle represents a bend to the right, i.e. towards negative $x$ values. For sector bending magnets, the bend radius is given by $\rho=L / \alpha$, and for rectangular bending magnets it has the value $\rho=L /(2 \sin (\alpha / 2))$. If the magnet is rotated about the $s$-axis by an angle $\psi, R$ and $S$ are transformed by

$$
\bar{R}=T R, \quad \bar{S}=T S T^{-1}
$$

where $T$ is the rotation matrix

$$
T=\left(\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right)
$$

The special value $\psi=\pi / 2$ represents a bend down.

### 1.3.3 Rotation of the Reference System

For a rotation of the reference system by an angle $\psi$ about the beam ( $s$ ) axis:

$$
S=\left(\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right)
$$

while for a rotation of the reference system by angle $\theta$ about the vertical ( $y$ ) axis:

$$
S=\left(\begin{array}{ccc}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{array}\right)
$$

In both cases the displacement $R$ is zero.

### 1.3.4 Elements which do not Change the Local Reference

MARKER elements do not affect the reference orbit. They are ignored for geometry calculations.

### 1.4 Sign Conventions for Magnetic Fields

The MAD program uses the following Taylor expansion [5] for the field on the mid-plane ( $y=0$ ):

$$
B_{y}(x, 0)=\sum_{n=0}^{\infty} \frac{B_{n} x^{n}}{n!}
$$

Note the factorial in the denominator. The field coefficients have the following meaning:
$B_{0} \quad$ Dipole field, with a positive value in the positive $y$ direction; a positive field bends a positively charged particle to the right.
$B_{1} \quad$ Quadrupole coefficient $B_{1}=\partial B_{y} / \partial x$; a positive value corresponds to horizontal focussing of a positively charged particle.
$B_{2} \quad$ Sextupole coefficient $B_{2}=\partial^{2} B_{y} / \partial x^{2}$.
$B_{3} \quad$ Octupole coefficient $B_{3}=\partial^{3} B_{y} / \partial x^{3}$.
Using this expansion and the curvature $h$ of the reference orbit, the longitudinal component of the vector potential to order 4 is:

$$
\begin{aligned}
A_{s}= & +B_{0}\left(x-\frac{h x^{2}}{2(1+h x)}\right) \\
& +B_{1}\left(\frac{1}{2}\left(x^{2}-y^{2}\right)-\frac{h}{6} x^{3}+\frac{h^{2}}{24}\left(4 x^{4}-y^{4}\right)+\ldots\right) \\
& +B_{2}\left(\frac{1}{6}\left(x^{3}-3 x y^{2}\right)-\frac{h}{24}\left(x^{4}-y^{4}\right)+\ldots\right)+B_{3}\left(\frac{1}{24}\left(x^{4}-6 x^{2} y^{2}+y^{4}\right)+\ldots\right)+\ldots
\end{aligned}
$$

Taking curl $A_{s}$ in curvilinear coordinates the field components can be computed as

$$
\left.\begin{array}{rl}
B_{x}(x, y)= & +B_{1}\left(y+\frac{h^{2}}{6} y^{3}+\ldots\right) \\
& +B_{2}\left(x y-\frac{h^{3}}{6} y^{3}+\ldots\right) \\
& +B_{3}\left(\frac{1}{6}\left(3 x^{2} y-y^{3}\right)+\ldots\right)+\ldots \\
B_{y}(x, y)= & +B_{0} \quad \\
& +B_{1}\left(x-\frac{h}{2} y^{2}+\frac{h^{2}}{2} x y^{2}+\ldots\right) \\
& +B_{2}\left(\frac{1}{2}\left(x^{2}-y^{2}\right)-\frac{h}{2} x y^{2}+\ldots\right)
\end{array}\right)+B_{3}\left(\frac{1}{6}\left(x^{3}-3 x y^{2}\right)+\ldots\right)+\ldots .
$$

It can be easily verified that $\operatorname{curl} \vec{B}$ and $\operatorname{div} \vec{B}$ are both zero to the order of the $B_{3}$ term. Introducing the magnetic rigidity $B \rho$, the multipole coefficients are computed as $K_{n}=e B_{n} / p_{0}=B_{n} / B \rho$.

### 1.5 Variables Used in MAD

For each variable the physical units are listed in square brackets.

### 1.5.1 Canonical Variables Describing Orbits

MAD uses the following canonical variables to describe the motion of particles:

X
PX Horizontal canonical momentum $p_{x}$ of the (closed) orbit referred to the ideal orbit, divided by the reference momentum: $\mathrm{PX}=p_{x} / p_{0},[1]$.

Y

Vertical canonical momentum $p_{y}$ of the (closed) orbit referred to the ideal orbit, divided by the reference momentum: $\mathrm{PY}=p_{y} / p_{0},[1]$.

Velocity of light times the negative time difference with respect to the reference particle: $\mathrm{T}=-c \Delta t,[\mathrm{~m}]$. A positive T means that the particle arrives ahead of the reference particle.

PT
Energy error, divided by the reference momentum times the velocity of light: PT $=$ $\delta=\Delta E / p_{0} c,[1]$. This value is only non-zero when synchrotron motion is present. It describes the deviation of the particle from the orbit of a particle with the momentum error DELTAP.

DELTAP
Difference of the reference momentum and the design momentum, divided by the reference momentum: DELTAP $=\delta_{s}=\Delta p_{s} / p_{0}$, [1]. This quantity is used to normalize all element strengths. See Chapter 15.1.1 for explanation.

The independent variable is:
S
Arc length along the reference orbit, [m].
In the limit of fully relativistic particles $(v=c, E=p c)$, the variables T, PT used here agree with $s, \delta$ used in TRANSPORT [6]. This means that T becomes the negative path length difference, while PT becomes the fractional momentum error. The reference momentum $p_{0}$ must be constant in order to keep the system canonical.

### 1.5.2 Normalised Variables and other Derived Quantities

XN
The normalised horizontal displacement $x_{n}=\Re E_{1}^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
PXN The normalised horizontal transverse momentum $p_{x n}=\Im E_{1}^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
WX The horizontal Courant-Snyder invariant $W_{x}=x_{n}^{2}+p_{x n}^{2}\left[\mathrm{~m}^{-1}\right]$.
PHIX The horizontal phase $\phi_{x}=-\arctan \left(p_{x n} / x_{n}\right) / 2 \pi[1]$.
YN
The normalised vertical displacement $y_{n}=\Re E_{2}^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
PYN The normalised vertical transverse momentum $p_{Y x n}=\Im E_{2} 1^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
WY The vertical Courant-Snyder invariant $W_{y}=y_{n}^{2}+p_{y n}^{2}\left[\mathrm{~m}^{-1}\right]$.
PHIY The vertical phase $\phi_{y}=-\arctan \left(p_{y n} / y_{n}\right) / 2 \pi$.
The normalised vertical displacement $t_{n}=\Re E_{3}^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
PTN The normalised vertical transverse momentum $p_{t n}=\Im E_{3}^{T} S Z\left[\mathrm{~m}^{-1 / 2}\right]$.
$W T \quad$ The longitudinal invariant $W_{y}=t_{n}^{2}+p_{t n}^{2}\left[\mathrm{~m}^{-1}\right]$.
PHIT The vertical phase $\phi_{t}=-\arctan \left(p_{t n} / t_{n}\right) / 2 \pi[1]$.
in the above formulas $Z$ is the phase space vector

$$
Z=\left(\begin{array}{l}
x \\
p_{x} \\
y \\
p_{y} \\
t \\
p_{t}
\end{array}\right)
$$

the matrix $S$ is the "symplectic unit matrix"

$$
S=\left(\begin{array}{cccccc}
0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0
\end{array}\right),
$$

and the vectors $E_{i}$ are the three complex eigenvectors.

### 1.5.3 Linear Lattice Functions (Optical Functions)

Several MAD commands refer to linear lattice functions. Since MAD uses the canonical momenta $p_{x}, p_{y}$ instead of the slopes $x^{\prime}, y^{\prime}$, their definitions differ slightly from those in Reference [10]. The linear lattice functions are known to MAD under the following names:

BETX Amplitude function $\beta_{x},[\mathrm{~m}]$.
ALFX $\quad$ Correlation function $\alpha_{x}=-(1 / 2)\left(\partial \beta_{x} / \partial s\right),[1]$.
MUX Phase function $\mu_{x},[2 \pi]$.
DX Dispersion of $x$, i.e. $D_{x}=\partial x / \partial \delta,[\mathrm{m}]$.
DPX Dispersion of $p_{x} / p_{0}$, i.e. $D_{p_{x}}=\left(1 / p_{0}\right)\left(\partial p_{x} / \partial \delta\right),[1]$.
BETY Amplitude function $\beta_{y},[\mathrm{~m}]$.
ALFY $\quad$ Correlation function $\alpha_{y}=-(1 / 2)\left(\partial \beta_{y} / \partial s\right),[1]$.
MUY Phase function $\mu_{y},[2 \pi]$.
DY $\quad$ Dispersion of $y$, i.e. $D_{y}=\partial y / \partial \delta,[\mathrm{m}]$.
DPY
Dispersion of $p_{y} / p_{0}$, i.e. $D_{p_{y}}=\left(1 / p_{0}\right)\left(\partial p_{y} / \partial \delta\right),[1]$.

### 1.5.4 Chromatic Functions

Several MAD commands refer to the chromatic functions. Since MAD uses the canonical momenta $p_{x}, p_{y}$ instead of the slopes $x^{\prime}, y^{\prime}$, their definitions differ slightly from those in Reference [24]. The chromatic functions are known to MAD under the following names:

WX $\quad$ Chromatic amplitude function $W_{x}=\sqrt{a_{x}^{2}+b_{x}^{2}}$, [1], where

$$
b_{x}=\frac{1}{\beta_{x}} \frac{d \beta_{x}}{d \delta}, \quad a_{x}=\frac{d \alpha_{x}}{d \delta}-\frac{\alpha_{x}}{\beta_{x}} \frac{d \beta_{x}}{d \delta} .
$$

PHIX
Chromatic phase function $\Phi_{x}=\arctan \left(a_{x} / b_{x}\right),[2 \pi]$.
DMUX $\quad$ Chromatic derivative of phase function $d \mu_{x} / d \delta,[2 \pi]$.
DDX Chromatic derivative of dispersion $d D_{x} / d \delta=d^{2} x / d \delta^{2},[\mathrm{~m}]$.
DDPX
Chromatic derivative of dispersion $d D_{p_{x}} / d \delta=\left(1 / p_{0}\right)\left(d^{2} p_{x} / d \delta^{2}\right),[1]$.

$$
b_{y}=\frac{1}{\beta_{y}} \frac{d \beta_{y}}{d \delta}, \quad a_{y}=\frac{d \alpha_{y}}{d \delta}-\frac{\alpha_{y}}{\beta_{y}} \frac{d \beta_{y}}{d \delta} .
$$

PHIY
DMUY
DDY
DDPY

Chromatic phase function $\Phi_{y}=\arctan \left(a_{y} / b_{y}\right),[2 \pi]$.
Chromatic derivative of phase function $d \mu_{y} / d \delta,[2 \pi]$.
Chromatic derivative of dispersion $d D_{y} / d \delta=d^{2} x / d \delta^{2},[\mathrm{~m}]$.
Chromatic derivative of dispersion $d D_{p_{y}} / d \delta=\left(1 / p_{0}\right)\left(d^{2} p_{x} / d \delta^{2}\right),[1]$.

### 1.5.5 Variables in the TUNES Tables

The TUNES option of the TWISS command produces a table which contains the following variables:
DELTAP Energy difference, divided by the reference momentum times the velocity of light: DELTAP $=\delta_{s}=\Delta E / p_{0} c$, [1].

ALFA The momentum compaction $\alpha=\frac{\Delta C / C}{\delta}[1]$.
GAMMATR The transition energy $\gamma_{t r}=E_{t r} / m_{0} c^{2}[1]$.
The horizontal tune [1].
QY
The vertical tune [1].
The horizontal chromaticity $\partial Q_{x} / \partial \delta$ [1].
The vertical chromaticity $\partial Q_{y} / \partial \delta[1]$.
XRMS
The horizontal r.m.s. closed orbit deviation [m].
YRMS The vertical r.m.s. closed orbit deviation [m].
XMAX The absolute maximum of horizontal closed orbit deviation [m].
YMAX The absolute maximum of vertical closed orbit deviation [m].
BXMAX $\quad$ The largest horizontal $\beta_{x}[\mathrm{~m}]$.
BYMAX $\quad$ The largest vertical $\beta_{y}[\mathrm{~m}]$.
DXMAX The absolutely largest horizontal dispersion [m].
DYMAX The absolutely largest vertical dispersion [m].

### 1.5.6 Variables in the TRACK Table

The command RUN writes tables with the following variables:

X
PX

Y

Horizontal position $x$ of the orbit, referred to the ideal orbit [ m ].
Horizontal canonical momentum $p_{x}$ of the orbit referred to the ideal orbit, divided by the reference momentum: $\mathrm{PX}=p_{x} / p_{0},[1]$.

Vertical position $y$ of the orbit, referred to the ideal orbit [m].

Vertical canonical momentum $p_{y}$ of the orbit referred to the ideal orbit, divided by the reference momentum: $\mathrm{PY}=p_{y} / p_{0},[1]$.

T
Velocity of light times the negative time difference with respect to the reference particle: $\mathrm{T}=-c \Delta t,[\mathrm{~m}]$. A positive T means that the particle arrives ahead of the reference particle.

DELTAP Energy difference, divided by the reference momentum times the velocity of light: DELTAP $=\delta=\Delta p / p_{s} c,[1]$.

### 1.6 Physical Units Used

Throughout the computations MAD uses international (SI, Système International) units. These units are summarised in Table 1.6.

Table 1.1: Physical Units used by MAD

| Quantity | Unit |
| :--- | :--- |
| Length | m (metres) |
| Angle | rad (radians) |
| Quadrupole coefficient | $\mathrm{m}^{-2}$ |
| Multipole coefficient, 2n poles | $\mathrm{m}^{-n}$ |
| Electric voltage | MV (Megavolts) |
| Electric field strength | $\mathrm{MV} / \mathrm{m}$ |
| Frequency | MHz (Megahertz) |
| Phase angles | multiples of $2 \pi$ |
| Particle energy | GeV |
| Particle mass | $\mathrm{GeV} / \mathrm{c}^{2}$ |
| Particle momentum | $\mathrm{GeV} / \mathrm{c}$ |
| Beam current | $\mathrm{A} \mathrm{(Ampères)}$ |
| Particle charge | e (elementary charges) |
| Impedances | $\mathrm{M} \Omega$ (Megohms) |
| Emittances | $\pi \cdot \mathrm{m}$ |
| RF power | MW (Megawatts) |
| Higher mode loss factor | $\mathrm{V} / \mathrm{pc}$ |

### 1.7 Wild Card Patterns

Some commands allow selection of items via "wild-card" strings. Such a pattern string must be enclosed in single or double quotes; and the case of letters is significant. The meaning of special characters is similar to the UNIX grep utility:

Stands for an arbitrary character,
[ $c_{1} c_{2} c_{3} \ldots$ ] Stands for one character belonging to the string contained in brackets (example: [abc] means one of $a, b, c$ ).
[ $\left.c_{1}-c_{2} c_{3}-c_{4} \ldots\right]$ Stands for ranges of characters (example: [a-zA-Z] means any letter).
*
Allows zero or more repetitions of the preceding item (example: [A-Z]* means zero or more upper-case letters).

All other characters stand for themselves. Examples:

```
REMOVE,PATTERN="D.."
SAVE,PATTERN="K.*QD.*\.R1"
```

The first command removes all elements whose names have exactly three characters and begin with the letter $D$. The second command saves definitions beginning with the letter $K$, containing the string QD, and ending with the string .R1. The two occurrences of .* each stand for an arbitrary number (including zero) of any character, and the occurrence $\backslash$. stands for a literal period.

### 1.8 Save Sequence as a Line, SAVELINE Command

A representation of the range set in the latest USE can be saved as an equivalent LINE definition using the command

```
SAVELINE,NAME=line-name, FILENAME=filename
```

where
NAME is the name to be given on the generated LINE definition,
FILENAME is the name of the file to be written.
The currently active range of the latest USE command will be written together with all parameter and element definitions. The output may contain a lot of irrelevant information, as no attempt is made to output only the definitions needed to regenerate an equivalent machine.


Figure 1.1: Local Reference System


Figure 1.2: Global Reference System

## Chapter 2. Commands and Statements

Table 2.1: Utility Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| HELP | Help on command names | 2.2 .1 |
| SHOW | Help on defined names | 2.2 .2 |
| TITLE | Define page header for output | 2.5 |
| STOP | End program run | 2.6 |
| OPTION | Set command options | 2.7 |
| $:=$ | Define parameter dependencies | 2.8 .1 |
| SET | Set parameter value | 2.8 .2 |
| VALUE | Show parameter values | 2.8 .3 |
| SYSTEM | Execute operating system command | 2.9 |

Input for MAD is free format. Blank characters or blank input lines do not affect program execution. Input lines are normally printed on the ECHO file as soon as they are read, but this feature can be turned off for long input files. In the following descriptions, words in lower case stand for syntactic units which are to be replaced by actual text. UPPER CASE is used for keywords or names. These must be entered as shown. Items enclosed in braces ( $\{\ldots\}$ ) may be repeated any number of times, including zero times. A vertical bar (|) represents an alternative.

### 2.1 Statements, Attributes, Options, Comments

The input file consists of a sequence of commands, also known as statements. Normally a statement occupies a single input line. Several statements may be placed on the same line, if they are separated by semicolons (;). A longer statement can be continued on several input lines. All but the last line of the statement are then terminated by an ampersand (\&). Blank input lines, or input lines beginning with an exclamation mark are accepted as comment lines. When an ampersand (\&) or an exclamation mark (!) is found on an input line, the remaining characters of the line are skipped, After an ampersand (\&) MAD expects continuation of the statement on the next input line, while an exclamation mark (!) terminates the statement. An input line cannot be longer than 80 characters; the length of one command is limited at 4000 non-blank characters.

The general format for a command is

```
label: keyword {,attribute}
```

It has three parts:
label Gives a name to the stored command. A label is required for a definition statement.
keyword Identifies the action desired.
attribute Most commands require attributes for their operation. These are normally entered in the form

```
attribute-name=attribute-value
```

The attribute-name selects the attribute, and attribute-value gives it a value.

The format of label and keyword is described in Section 2.3.
In many cases the input can be abbreviated by omitting attribute-name and the equals sign. The attribute-values must then be entered in the order in which attributes appear in the command dictionary. For some attributes it is sufficient to enter the name only. The attribute is then given a default value taken from the command dictionary. Example: TILT attribute for various magnets.

Whenever this makes sense, an attribute can be left out altogether. To avoid overloading the syntax descriptions, this usage is not indicated.

The command dictionary also defines one of the following types for each command attributes:

- Name attribute (see Section 2.4.1),
- String attribute (see Section 2.4.2),
- Logical attribute (see Section 2.4.3),
- Integer attribute (see Section 2.4.4),
- Real expression (see Section 2.4.5),
- Deferred expression (see Section 2.4.8),
- Constraint (see Section 2.4.9),
- Variable name (see Section 2.4.10),
- Line definition (see Section 4.5),
- Range selection (see Section 4.7),

When a command is entered with a label, MAD keeps it in memory. This allows repeated execution of the same command by just entering its label. If the label of such a command appears together with new attributes, the attributes will be replaced first:

```
QF: QUADRUPOLE,L=1,K1=0.01 ! first definition of QF
QF,L=2 ! redefinition of QF, new length
TW1: TWISS,BETX=1,BETY=1 ! first execution of TW1
    ! with BETX=1, BETY=1
TW1,BETX=2,BETY=3 ! second execution of TW1
    ! with BETX=2, BETY=3
```


### 2.2 Getting Help

### 2.2.1 HELP Command

A user who is uncertain about the attributes of a command should try the HELP command

```
HELP, keyword
```

The program then prints the list of attributes for the command keyword with their type and, if they exist, their defaults and limits. HELP entered alone lists all available commands. Examples:

```
HELP ! List all valid keywords
HELP,TWISS ! List attributes of "TWISS"
```


### 2.2.2 SHOW Command

The SHOW statement displays the current values of a command, element, line, or parameter.
SHOW, name

It displays the named command or definition on the ECHO file in the same format as the original input statement used to create the item viewed. SHOW entered alone lists all known definitions. Examples:

```
SHOW ! Show all defined names
SHOW,QD ! Show definition of QD
```


### 2.3 Identifiers or Labels

A label begins with a letter, followed by up to fifteen letters, digits, decimal points (.), primes ('), or underscores (_). Characters beyond the sixteenth are dropped, and the resulting sequence must be unique. Other special characters are allowed, if they are enclosed in single or double quotes. It makes no difference which type of quotes is used, as long as the same are used at both ends. The preferred form is double quotes.

A label may refer to a keyword, an element, a beam-line, a parameter, etc. Once it is defined, a reference can be abbreviated by dropping trailing characters, as long as the abbreviation fits only one defined label. Note that abbreviations may introduce ambiguities which cannot always be resolved correctly by MAD.

MAD maintains two name spaces, one for keywords, and one for user-defined names. Thus no conflict arises if a user-defined name is the same as a predefined keyword.

### 2.4 Command Attributes

### 2.4.1 Name Attributes

A name attribute often selects one of a set of options:
RUN, METHOD=TRANSPORT ! track by the "TRANSPORT" method

It may also refer to a user-defined object:

```
ARCHIVE,TABLE=TWISS ! archive the table "TWISS"
```


### 2.4.2 String Attributes

A string attribute makes alphanumeric information available, e.g. a title or a file name. Examples:

```
TITLE,"This is a title for the program run"
POOLDUMP,FILENAME="pool.dump.file"
```


### 2.4.3 Logical Attributes

Many commands in MAD require the setting of logical values (flags) to represent the on/off state of an option. A logical value flag can be set in several ways:

```
flag | flag=.YES. | flag=.TRUE. | flag=.T. | flag=.ON.
```

It can be reset by any of the following:

Example:

```
USE,...,SYMM ! the beam line is symmetric
```

The default for a logical flag is always .FALSE..

### 2.4.4 Integer Attributes

An integer attribute usually denotes a count. Example:

```
USE,...,SUPER=4 ! there are four superperiods
```


### 2.4.5 Real Expressions

To facilitate the definition of interdependent quantities, any real value can be entered as an arithmetic expression. When a value used in an expression is redefined by the user or changed in a matching process, the expression is reevaluated. Expression definitions may be entered in any order. MAD evaluates them in the correct order before it performs any computation. At evaluation time all operands used must have values assigned.

### 2.4.6 Operators in Arithmetic Expressions

An expression can be formed using the following operators:

- Arithmetic operators:

| + | Addition, |
| :--- | :--- |
| - | Subtraction, |
| * | Multiplication, |
| $/$ | Division, |
| - | Exponentiation. |

- Ordinary functions:

| $\operatorname{SQRT}(X)$ | Square root, |
| :--- | :--- |
| $\operatorname{LOG}(X)$ | Logarithm, |
| $\operatorname{EXP}(X)$ | Exponential, |
| $\operatorname{SIN}(X)$ | Trigonometric sine, |
| $\operatorname{COS}(X)$ | Trigonometric cosine, |
| $\operatorname{TAN}(X)$ | Trigonometric tangent, |
| $\operatorname{ASIN}(X)$ | Arc sine,, |
| $\operatorname{ABS}(X)$ | Absolute value, |
| $\operatorname{MAX}(X, Y)$ | Maximum of two values,, |
| $\operatorname{MIN}(X, Y)$ | Minimum of two values. |

- Random number generators:

RANF () Random number, uniformly distributed in [0,1],
GAUSS() Random number, Gaussian distribution with unit standard deviation,

TGAUSS(X) Random number, Gaussian distribution with unit standard deviation, truncated at X standard deviations,

USERO() Random number, user-defined distribution without arguments,
USER1 ( X ) Random number, user-defined distribution with one argument,
USER2 ( $\mathrm{X}, \mathrm{Y}$ ) Random number, user-defined distribution with two arguments.
Parentheses indicate operator precedence if required. Constant sub-expressions are evaluated immediately, and the result is stored as a constant. Exponentiation is not directly available. However, it may be performed by the identity $A^{B}=\exp (B * \log (A))$.

In an ordinary expression a random generator is only permitted if it has no variable arguments. A random number is then generated at definition time and stored as a constant. Example:

```
D:DRIFT,L=0.01*RANF() ! a drift space with random length,
    ! generated at definition time.
E:DRIFT,L=USER1(X) ! not permitted, if X is a variable
```


### 2.4.7 Operands in Arithmetic Expressions

An expression may contain the following operands:

- Literal constants: Numerical values are entered like FORTRAN constants. Real values are accepted in INTEGER or REAL format. The use of a decimal exponent, marked by the letter D or E, is permitted. Examples:

```
1, 10.35, 5E3, 314.1592E-2
```

- Symbolic constants: MAD recognises the mathematical and physical constants listed in Table 2.2. Their names should not be used for user-defined labels.

Additional symbolic constants may be defined to simplify their repeated use in statements and expressions. The Constant command

```
label: CONSTANT=constant-expression
```

defines a constant with the name label. Label must be unique. An existing symbolic constant can be redefined, but it cannot change in a matching procedure. A reference to a constant is immediately replaced by its value. Thus redefinition of a constant does not affect any preceding definitions. Example:

```
IN: CONSTANT=0.0254
```

Table 2.2: Symbolic Constants

| symbol | name | Value used 0 | unit |
| :--- | :--- | :---: | :--- |
| $\pi$ | PI | 3.1415926535898 | 1 |
| $2 \pi$ | TWOPI | 6.2831853071796 | 1 |
| $180 / \pi$ | DEGRAD | 57.295779513082 | $\circ / \mathrm{rad}$ |
| $\pi / 180$ | RADDEG | 0.017453292519943 | $\mathrm{rad} /{ }^{\circ}$ |
| $e$ | E | 2.7182818284590 | 1 |
| $m_{e}$ | EMASS | $0.51099906^{*} 10^{-3}$ | GeV |
| $m_{p}$ | PMASS | 0.93827231 | GeV |
| $c$ | CLIGHT | $2.99792458^{*} 10^{8}$ | $\mathrm{~m} / \mathrm{s}$ |

- Parameter labels: Often a set of numerical values depends on a common variable parameter. Such a parameter must be defined according to Section 2.8.1. When its name is used in an expression, MAD uses the current value of the parameter. Example:

```
        X:=1.0
D1: DRIFT,L=X
D2: DRIFT,L=2.0-X
```

When the value of $X$ is changed, the lengths of the drift spaces are recalculated as $X$ and $2-X$ respectively.

- Element or command attributes: In arithmetic expressions the attributes of physical elements or commands can occur as operands. They are named respectively by

```
element-name[attribute-name]
command-name[attribute-name]
```

Values are assigned to attributes in element definitions or commands. Example:

```
D1: DRIFT,L=1.0
D2: DRIFT,L=2.0-D1[L]
```

D1 [L] refers to the length $L$ of the drift space $D 1$.

### 2.4.8 Deferred Expressions and Random Values

The definition of random machine imperfections requires evaluation of expressions containing random functions. These are not evaluated like other expressions before a command begins execution, but sampled as required from the distributions indicated when errors are generated. Such an expression is known as a deferred expression. Its value cannot occur as an operand in another expression. Example:

```
ERROR: EALIGN,range,DX=SIGMA*GAUSS()
```

All elements in range are assigned independent random displacements sampled from a Gaussian distribution with standard deviation SIGMA. The quantity ERROR[DX] must not occur as an operand in another expression.

### 2.4.9 Constraints

In matching it is desired to specify equality constraints, as well as lower and upper limits for a quantity. MAD accepts the following forms of constraints:

```
name=expression ! equality constraint
name<expression ! upper limit
name>expression ! lower limit
name<expression,name>expression ! both upper and lower limit
for the same name
```


### 2.4.10 Variable Names

A variable name can have one of the formats:

```
parameter-name
command-name[attribute-name]
element-name[attribute-name]
```

The first format refers to the value of the global parameter parameter-name (see Section 2.8.1), the second format refers to the attribute attribute-name of the command command-name or element element-name respectively.

### 2.5 TITLE Statement

The TITLE statement

```
TITLE,S=page-header
```

expects a string page-header, enclosed in single or double quotes, which will be used as a title for subsequent output pages. Before a TITLE statement is encountered, the page header is blank. It can be redefined at any time.

### 2.6 STOP Statement

The STOP statement

```
STOP
```

terminates execution of the program. Any statement following the STOP statement is ignored.

### 2.7 OPTION Statement

The OPTIONS command (former SETOPTS statement) controls global command execution:

```
OPTION, RESET,INTER,ECHO,TRACE,DOUBLE,VERIFY,WARN,DEBUG,&
    INFO,SYMPL,&
    KEYWORD=integer, COMMAND=integer,DEFINE=integer,&
    EXPRESS=integer,LINE=integer,TELL
```

The first seven logical flags activate or deactivate execution options:
RESET If true, all options are reset to their default values listed below before setting.
INTER Normally MAD determines itself whether it runs interactively. On some computer systems this may not be possible; the user may then set this flag manually.

ECHO Controls printing of an echo of input lines on the file ECHO.
TRACE When the TRACE option is on, MAD writes additional information on the ECHO file for each executable command. This information includes the command name, elapsed CPU time before and after the command, and the CPU time used for the command.

DOUBLE

VERIFY If this option is on, MAD gives a message for each undefined variable.

WARN If this option is turned off, MAD suppresses all warning messages.
INFO If this option is turned off, MAD suppresses all information messages.
SYMPL If this option is turned off, MAD suppresses matrix symplectification.
DEBUG This option is reserved for the programmer.
The next five integer flags control debugging output. Each of them can have four different values:
0 No output.
1 Output in "comprehensive" format.
2
Output by the ZEBRA dump routines.
$3 \quad$ Output in both formats.
The relevant output flags are:
KEYWORD Dump every new keyword definition.
COMMAND Dump every executable command before execution.
DEFINE Dump every new element or parameter definition.
EXPRESS Dump arithmetic expressions when they are decoded or evaluated.
LINE Dump every each new line definition.
One attribute controls the closed orbit finder:
COFACT The former parameter COFACT is now ignored.
The last attribute requests listing of the current settings:
TELL If true, the current settings are listed.
Example:
OPTION, -ECHO,TELL

Turns off the command echo print-out and lists the current settings. If contradicting options are entered, the last option given prevails. Example:

OPTION, ECHO , -ECHO
turns off the ECHO option. The default settings at program start-up time are listed in Table 2.3.

Table 2.3: Initial Defaults for command options

| option | setting | option | setting | option | setting | option | setting |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| INTER | on | ECHO | on | TRACE | off | DOUBLE | off |
| KEYWORD | 0 | COMMAND | 0 | DEFINE | 0 | EXPRESS | 0 |
| LINE | 0 |  |  |  |  |  |  |

### 2.8 Parameter Statements

### 2.8.1 Relations between Variable Parameters

A relation is established between variables by the statement

```
parameter-name:=expression
```

It creates a new parameter parameter-name and discards any old parameter with the same name. Its value depends on all quantities occurring in expression on the right-hand side. Whenever an operand in expression changes, a new value is calculated. The definition may be thought of as a mathematical equation; but MAD is not able to solve the equation for a quantity on the right-hand side. Example:

```
GEV:=100
BEAM, ENERGY=GEV
```

Circular definitions are not allowed (but see Section 2.8.2):

```
X:=X+1 ! X cannot be equal to X+1
A:=B
B:=A ! A and B are equal, but of unknown value
```


### 2.8.2 Assignment to Parameters

A value is assigned to a parameter by the SET statement

```
SET,VARIABLE=parameter-name,VALUE=expression
```

This statement acts like a FORTRAN assignment. If the parameter parameter-name does not yet exist, it is created. Then the expression is evaluated, and the result is assigned to the parameter parameter-name. Finally the expression is discarded. Therefore a sequence like the following is permitted:

```
... ! some definitions
USE,line
X:=0 ! create parameter X with value zero
DO,TIMES=10
    TWISS
    SET,X,X+.01
ENDDO
```


### 2.8.3 Output of Parameters

The VALUE statement

```
VALUE,VALUE=expression{,expression}
```

evaluates up to five expressions using the most recent values of any operands and prints the results on the ECHO file. Example:

```
P1:=5
P2:=7
VALUE,P1*P2-3
```

After echoing the command, this prints:
AAVALU. Value of expression "P1*P2-3" is 32.00000000
The main use of this commands is for printing a quantity which depends on matched attributes. It allows use of MAD as a programmable calculator. One may also tabulate functions.

### 2.9 SYSTEM: Execute Operating System Command

During an interactive MAD session the command SYSTEM allows to execute operating system commands. After execution of the system command, successful or not, control returns to MAD. At present this command is only available under UNIX or VM/CMS. Its format is:
SYSTEM,"string"
where string is a valid operating system command.

### 2.9.1 SYSTEM Command under UNIX

Most UNIX commands can be issued directly. Example:
SYSTEM,"ls -1"
causes a listing of the current directory in long form on the terminal.

### 2.9.2 SYSTEM Command under VM/CMS

If the system command refers to an EXEC file the string must begin with the word EXEC. Synonyms or abbreviations are not always accepted. It is recommended to use standard CMS command names and to spell them out in full. Examples:

```
SYSTEM,"ERASE TESTDATA MAD A"
SYSTEM,"EXEC FILELIST * * D"
SYSTEM,"XEDIT TESTDATA MAD A"
```


### 2.10 COMMENT/ENDCOMMENT Statements

Commands bracketed between COMMENT and ENDCOMMENT are not executed, but skipped. These commands may be nested. Example:

```
EMIT ! executed
COMMENT
SURVEY ! not executed (nesting level 1)
COMMENT
SURVEY ! not executed (nesting level 2)
ENDCOMMENT
TWISS ! still not executed (nesting level 1)
ENDCOMMENT
TWISS ! executed again
```


## Chapter 3. Physical Elements and Markers

Table 3.1: Element Definition Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| MARKER | Marker for beam observation | 3.2 |
| DRIFT | Drift space | 3.3 |
| SBEND | Sector bending magnet | 3.4 |
| RBEND | Rectangular bending magnet | 3.4 |
| QUADRUPOLE | Quadrupole | 3.5 |
| SEXTUPOLE | Sextupole | 3.6 |
| OCTUPOLE | Octupole | 3.7 |
| MULTIPOLE | Thin multipole | 3.8 |
| SOLENOID | Solenoid | 3.9 |
| HKICKER | Horizontal orbit corrector | 3.10 |
| VKICKER | Vertical orbit corrector | 3.10 |
| KICKER | Corrector for both planes | 3.10 |
| RFCAVITY | RF cavity | 3.11 |
| ELSEPARATOR | electrostatic separator | 3.12 |
| HMONITOR | Horizontal orbit position monitor | 3.13 |
| VMONITOR | Vertical orbit position monitor | 3.13 |
| MONITOR | Orbit position monitor (both planes) | 3.13 |
| INSTRUMENT | Space for beam instrumentation | 3.13 |
| ECOLLIMATOR | Elliptic collimator | 3.14 |
| RCOLLIMATOR | Rectangular collimator | 3.14 |
| YROT | Rotation about vertical axis | 3.15 |
| SROT | Rotation about longitudinal axis | 3.15 |
| BEAMBEAM | Beam-beam interaction | 3.16 |
| MATRIX | Arbitrary matrix | 3.17 |
| LUMP | Concatenation of elements | 3.20 |

### 3.1 Input Format

All physical elements are defined by statements of the form

$$
\text { label: keyword [,TYPE=name] \{,attribute\} }
$$

Example:
QF: QUADRUPOLE,TYPE=MQ,L=1.8,K1=0.015832
where
label A name to be given to the element (in the example QF),
keyword An element type keyword (in the example QUADRUPOLE),
TYPE A label to be attached to the element. It denotes the "engineering type" as defined documentlabelin earlier versions of MAD, and may be used for selection of elements in various commands like error definitions. (in the example MQ).
attribute Normally has the form

Attribute-name selects the attribute, as defined for the element type keyword (in the example L and K1), and attribute-value gives it a value (in the example 1.8 and 0.015832 ).

Omitted attributes are assigned a default value, normally zero. For some attributes it is permitted to enter their name only. In this case the attributes are assigned a special default value (Example: TILT angles for magnets). If such usage is allowed, it is indicated with the respective attribute.

### 3.2 Marker Definitions

```
label: MARKER,TYPE=name
```

The simplest element which can occur in a beam line is the MARKER. It has no effect on the beam, but it allows one to identify a position in the beam line, for example to apply a matching constraint. A marker has only the TYPE attribute: Example:

M27: MARKER,TYPE=MM

### 3.3 Drift Space

```
label: DRIFT,TYPE=name,L=real
```

A DRIFT space has one real attribute:
L The drift length (default: 0 m )
Examples:

```
DR1: DRIFT,L=1.5
DR2: DRIFT,L=DR1[L],TYPE=DRF
```

The length of DR2 will always be equal to the length of DR1. The reference system for a drift space is shown in Figure 3.1.


Figure 3.1: Reference System for Straight Beam Elements

### 3.4 Bending Magnets

Two different type keywords are recognised for bending magnets, they are distinguished only by the reference system used:


Figure 3.2: Reference System for a Rectangular Bending Magnet; the signs of pole-face rotations are positive as shown.

RBEND Rectangular bending magnet (Reference see Figure 3.2),
SBEND $\quad$ Sector bending magnet (Reference see Figure 3.3).

```
SBEND, TYPE=name,L=real,ANGLE=real,K1=real,E1=real,E2=real,&
    TILT=real,K2=real,H1=real,K2=real,HGAP=real,FINT=real,&
    K3=real
RBEND, TYPE=name,L=real,ANGLE=real,K1=real,E1=real,E2=real,&
    TILT=real,K2=real,H1=real,K2=real,HGAP=real,FINT=real,&
    K3=real
```

For both types, the following first-order attributes are permitted:
L The length of the magnet (default: 0 m ). For a rectangular magnet the length is measured along a straight line, while for a sector magnet it is the arc length of the reference orbit.

ANGLE The bend angle (default: 0 rad). A positive bend angle represents a bend to the right, i.e. towards negative $x$ values.

K1
The quadrupole coefficient $K_{1}=(1 / B \rho)\left(\partial B_{y} / \partial x\right)$. The default is $0 \mathrm{~m}^{-2}$. A positive quadrupole strength implies horizontal focussing of positively charged particles.

The rotation angle for the entrance pole face (default: 0 rad ).
E2
FINT
The rotation angle for the exit pole face (default: 0 rad ).
The field integral, see [5] and below. The default value is 0 .
HGAP
The half gap of the magnet (default: 0 m ).


Figure 3.3: Reference System for a Sector Bending Magnet; the signs of pole-face rotations are positive as shown.

The pole face rotation angles are referred to the magnet model (see Figure 3.2 and Figure 3.3). The quantities FINT and HGAP specify the finite extent of the fringe fields [5]. There they are defined as follows:

$$
K_{1}=\mathrm{FINT}=\int_{-\infty}^{\infty} \frac{B_{y}(s)\left(B_{0}-B_{y}(s)\right)}{g \cdot B_{0}^{2}} d s, \quad g=2 \cdot \mathrm{HGAP} .
$$

The default values of zero corresponds to the hard-edge approximation, i.e. a rectangular field distribution. For other approximations, enter the correct value of the half gap, and one of the following values for FINT:

| Linear drop-off of the field | $1 / 6$ |
| :--- | :--- |
| Clamped "Rogowski" fringing field | 0.4 |
| Un-clamped "Rogowski" fringing field | 0.7 |
| "Square-edged" non-saturating magnet | 0.45 |

Entering the keyword FINT alone sets the integral to 0.5 . This is a reasonable average of the above values. The magnet may be rotated about the longitudinal axis by use of the following parameter:

TILT The roll angle about the longitudinal axis (default: 0 rad, a positive bend angle then denotes a bend to the right). A vertical bend is defined by entering TILT with no value; this implies a roll of $\pi / 2 \mathrm{rad}$, i.e. a positive bend angle denotes a deflection down. A positive angle represents a clockwise rotation. The following second-order attributes are permitted:

K2
H1
H2
The sextupole coefficient $K_{2}=(1 / B \rho)\left(\partial^{2} B_{y}\right) /\left(\partial x^{2}\right)$ (default: $0 \mathrm{~m}^{-3}$ ).
The curvature of the entrance pole face (default: $0 \mathrm{~m}^{-1}$ ).
The curvature of the exit pole face (default: $0 \mathrm{~m}^{-1}$ ). A positive pole face curvature induces a negative sextupole component; i.e. for positive H 1 and H 2 the centres of curvature of the pole faces are placed inside the magnet.

One third-order parameter is accepted, but at present it is ignored:

Examples:

```
BR: RBEND,L=5.5,ANGLE=+0.001 ! Deflection to the right
BD: SBEND,L=5.5,ANGLE=+0.001,TILT ! Deflection down
BL: SBEND,L=5.5,ANGLE=-0.001 ! Deflection to the left
BU: SBEND,L=5.5,ANGLE=-0.001,TILT
! Deflection up
```


### 3.5 Quadrupole

```
label: QUADRUPOLE,TYPE=name,L=real,K1=real,TILT=real
```

A QUADRUPOLE has three real attributes:
L The quadrupole length (default: 0 m ).
K1 The quadrupole coefficient $K_{1}=(1 / B \rho)\left(\partial B_{y} / \partial x\right)$. The default is $0 \mathrm{~m}^{-2}$. A positive quadrupole strength implies horizontal focussing of positively charged particles.

TILT The roll angle about the longitudinal axis (default: 0 rad, i. e. a normal quadrupole). A skewed quadrupole is defined by entering TILT with no value; this implies a roll of $\pi / 4 \mathrm{rad}$ about the $s$-axis. A positive angle represents a clockwise rotation.

Example:
QF: QUADRUPOLE, L=1.5,K1=0.001

The reference system for a quadrupole is shown in Figure 3.1.

### 3.6 Sextupole

label: SEXTUPOLE,TYPE=name,L=real, K2=real,TILT=real

A SEXTUPOLE has three real attributes:
$\mathrm{L} \quad$ The sextupole length (default: 0 m ).
K2 The sextupole coefficient $K_{2}=(1 / B \rho)\left(\partial^{2} B_{y} / \partial x^{2}\right)\left(\right.$ default: $\left.0 \mathrm{~m}^{-3}\right)$.
TILT The roll angle about the longitudinal axis (default: 0 rad, i. e. a normal sextupole). A skewed sextupole is defined by entering TILT with no value; this implies a roll of $\pi / 6 \mathrm{rad}$ about the $s$-axis. A positive angle represents a clockwise rotation.

Example:

```
S: SEXTUPOLE,L=0.4,K2=0.00134
```

The reference system for a sextupole is shown in Figure 3.1.

### 3.7 Octupole

label: OCTUPOLE,TYPE=name, L=real,K3=real,TILT=real

An OCTUPOLE has three real attributes:
L The octupole length (default: 0 m ).
K3 The octupole coefficient $K_{3}=(1 / B \rho)\left(\partial^{3} B_{y} / \partial x^{3}\right)\left(\right.$ default: $\left.0 \mathrm{~m}^{-4}\right)$.
TILT The roll angle about the longitudinal axis (default: 0 rad, i. e. a normal octupole). A skewed octupole is defined by entering TILT with no value; this implies a roll of $\pi / 8 \mathrm{rad}$ about the $s$-axis. A positive angle represents a clockwise rotation.

Example:
03: OCTUPOLE, L=0.3, K3=0.543
The reference system for a octupole is shown in Figure 3.1. Octupoles are normally treated as thin lenses, except when tracking by Lie-algebraic methods.

### 3.8 General Thin Multipole

```
label: MULTIPOLE,TYPE=name,LRAD=real,&
K0L=real,T0=real,K1L=real,T1=real,&
K2L=real,T2=real,K3L=real,T3=real,&
K4L=real,T4=real,K5L=real,T5=real,&
K6L=real,T6=real,K7L=real,T7=real,&
K8L=real,T8=real,K9L=real,T9=real
```

A MULTIPOLE is thin lens of arbitrary order, including a dipole:
LRAD A fictitious length, which is only used to compute synchrotron radiation effects.
$\mathrm{KnL} \quad$ The multipole coefficient of order $n: K_{n} L=(L / B \rho)\left(\partial^{n} B_{y} / \partial x^{n}\right)$. (default: $\left.0 \mathrm{~m}^{-n}\right)$. This is the multipole coefficient integrated over the length of the multipole. The digit $n$ may take the values $0 \leq n \leq 9$. The number of poles of the component is $2 n+2$. The most important error components of quadrupoles are: K5L, the twelve-pole, and K9L, the twenty-pole. Superposition of several multipole components is permitted.

Tn
The roll angle for the multipole component of order $n$ about the $s$-axis (default: 0 rad, i. e. a normal multipole). A skewed multipole is defined by entering Tn with no value; this implies a roll of $\pi /(2 n+2)$ rad about the $s$-axis. A positive angle represents a clockwise rotation.

The multipole element has no length. Example:
M27: MULTIPOLE,K3L=0.0001,T3,K2L=0.0001
A multipole with no dipole component has no effect on the reference orbit, i.e. the reference system at its exit is the same as at its entrance. If it includes a dipole component, it has the same effect on the reference orbit as a dipole with zero length and deflection angle KOL.

### 3.9 Solenoid

label: SOLENOID,TYPE=name,L=real,KS=real

A SOLENOID has two real attributes:
$\mathrm{L} \quad$ The length of the solenoid (default: 0 m )
KS
The solenoid strength $B_{0} / B \rho$ (default: $0 \mathrm{rad} / \mathrm{m}$ ). For positive KS and positive particle charge, the solenoid field points in the direction of increasing $s$.

Example:
SOLO: SOLENOID, L=2., K=0.001
The reference system for a solenoid is shown in Figure 3.1.

### 3.10 Closed Orbit Correctors

Three types of closed orbit correctors are available:
HKICKER A corrector for the horizontal plane,
VKICKER A corrector for the vertical plane,
KICKER A corrector for both planes.

```
label: HKICKER, TYPE=name,L=real,KICK=real,TILT=real
label: VKICKER, TYPE=name,L=real,KICK=real,TILT=real
label: KICKER, TYPE=name,L=real,HKICK=real,VKICK=real,TILT=real
```

They have the following attributes:
L The length of the closed orbit corrector (default: 0 m ).
KICK The kick angle for either horizontal or vertical correctors. (default: 0 rad).
HKICK The horizontal kick angle for a corrector in both planes (default: 0 rad).
VKICK The vertical kick angle for a corrector in both planes (default: 0 rad).
TILT The roll angle about the longitudinal axis (default: 0 rad). A positive angle represents a clockwise rotation of the corrector.

A positive kick increases $p_{x}$ or $p_{y}$ respectively. Examples:

```
    HK1: HKICKER, KICK=0.001,TILT=RADDEG*10.0
    VK3: VKICKER, KICK=0.0005
    KHV: KICKER, HKICK=0.001,VKICK=0.0005
```

The first kicker is rotated about the longitudinal axis by 10 degrees. The reference system for a closed orbit corrector is shown in Figure 3.1.

### 3.11 RF Cavity

```
label: RFCAVITY,TYPE=name,L=real,VOLT=real,LAG=real,&
    HARMON=integer,BETRF=real,PG=real,&
    SHUNT=real,TFILL=real
```

An RFCAVITY has seven real attributes and one integer attribute:
L The length of the cavity (default: 0 m )
VOLT The peak RF voltage (default: 0 MV ). The effect of the cavity is
$\delta E=V O L T \sin \left(2 \pi\left(L A G-h f_{0} \Delta t\right)\right)$.
LAG The phase lag in multiples of $2 \pi$ (default: 0 ).
HARMON The harmonic number $h$ (no default). Note that the RF frequency is computed from the harmonic number and the revolution frequency $f_{0}$. The frequency attribute FREQ must no longer be used.

BETRF $\quad$ RF coupling factor (default: 0 ).
PG The RF power per cavity (default: 0 MW ).
SHUNT The relative shunt impedance (default: $0 \mathrm{M} \Omega / \mathrm{m}$ ).
TFILL The filling time of the cavity (default: $0 \mu \mathrm{~s}$ ).
A cavity requires the particle energy (ENERGY) and the particle charge (CHARGE) to be set by a BEAM command (see Section 5.2) before any calculations are performed. Example:

```
        BEAM,PARTICLE=ELECTRON,ENERGY=50.0
CAVITY: RFCAVITY,L=10.0,VOLT=150.0,LAG=0.0,HARMON=31320
```

The reference system for a cavity is shown in Figure 3.1.

### 3.12 Electrostatic Separator

label: ELSEPARATOR,TYPE=name, L=real, E=real,TILT=real
An ELSEPARATOR (electrostatic separator) has three real attributes:
L The length of the separator (default: 0 m ).
E The electric field strength (default: $0 \mathrm{MV} / \mathrm{m}$ ). A positive field increases $p_{y}$ for positive particles.

TILT The roll angle about the longitudinal axis (default: 0 rad ). A positive angle represents a clockwise rotation of the separator.

A separator requires the particle energy (ENERGY) and the particle charge (CHARGE) to be set by a BEAM command (see Section 5.2) before any calculations are performed. Example:

```
    BEAM,PARTICLE=POSITRON,ENERGY=50.0
SEP: ELSEPARATOR,L=5.0,E=0.5
```

The reference system for a separator is shown in Figure 3.1.

### 3.13 Beam Position Monitor

A beam monitor acts on the beam like a drift space. In addition it serves to record the beam position for closed orbit corrections. Four different types of beam position monitors are recognised:

HMONITOR Monitor for the horizontal beam position,
VMONITOR Monitor for the vertical beam position,
MONITOR Monitor for both horizontal and vertical beam position.
INSTRUMENT A place holder for any type of beam instrumentation. Optically it behaves like a drift space; it returns no beam observation. It represent a class of elements which is completely independent from drifts and monitors.

```
label: HMONITOR, TYPE=name,L=real
label: VMONITOR, TYPE=name,L=real
label: MONITOR, TYPE=name,L=real
label: INSTRUMENT, TYPE=name,L=real
```

A beam position monitor has one real attribute:
$\mathrm{L} \quad$ The length of the monitor (default: 0 m ). If the length is different from zero, the beam position is recorded in the centre of the monitor.

Examples:
MH: HMONITOR,L=1
MV: VMONITOR
The reference system for a monitor is shown in Figure 3.1.

### 3.14 Collimators (Aperture Definitions)

Two types of collimators are defined:
ECOLLIMATOR Elliptic aperture,
RCOLLIMATOR Rectangular aperture.

```
label: ECOLLIMATOR,TYPE=name,L=real,XSIZE=real,YSIZE=real
label: RCOLLIMATOR,TYPE=name,L=real,XSIZE=real,YSIZE=real
```

Either type has three real attributes:
L The collimator length (default: 0 m ).
XSIZE The horizontal half-aperture (default: unlimited).
YSIZE The vertical half-aperture (default: unlimited).
For elliptic apertures, XSIZE and YSIZE denote the half-axes respectively, for rectangular apertures they denote the half-width of the rectangle. Optically a collimator behaves like a drift space, but during tracking, it also introduces an aperture limit. The aperture is checked at the entrance. If the length is not zero, the aperture is also checked at the exit. The reference system for a collimator is shown in Figure 3.1. Example:

COLLIM: ECOLLIMATOR,L=0.5,XSIZE=0.01,YSIZE=0.005

### 3.15 Coordinate Transformations

### 3.15.1 Rotation About the Vertical Axis

```
label: YROT,TYPE=name,ANGLE=real
```

The element YROT rotates the reference system about the vertical ( $y$ ) axis. The reference system is shown in Figure 3.4. YROT has no effect on the beam, but it causes the beam to be referred to the new coordinate system

$$
x_{2}=x_{1} \cos \theta-s_{1} \sin \theta, \quad s_{2}=x_{1} \sin \theta+s_{1} \cos \theta
$$

It has one real attribute:
ANGLE The rotation angle $\theta$ (default: 0 rad ). It must be a small angle, i.e. an angle comparable to the transverse angles of the orbit.

A positive angle means that the new reference system is rotated clockwise about the local $y$-axis with respect to the old system. Example:

KINK: YROT, ANGLE $=0.0001$


Figure 3.4: Reference System for a Rotation Around the y-Axis

### 3.15.2 Rotation Around the Longitudinal Axis

```
label: SROT,TYPE=name,ANGLE=real
```

The element SROT rotates the reference system about the longitudinal ( $s$ ) axis. The reference system is shown in Figure 3.5. SROT has no effect on the beam, but it causes the beam to be referred to the new coordinate system

$$
x_{2}=x_{1} \cos \psi+y_{1} \sin \psi, \quad y_{2}=x_{1} \sin \psi+y_{1} \cos \psi
$$

It has one real attribute:
ANGLE $\quad$ The rotation angle $\psi$ (default: 0 rad)

A positive angle means that the new reference system is rotated clockwise about the $s$-axis with respect to the old system. Example:

ROLL1: SROT,ANGLE=PI/2.
ROLL2: SROT,ANGLE=-PI/2.
HBEND: SBEND,L=6.0,ANGLE=0.01
VBEND: LINE=(ROLL1, HBEND, ROLL2)
The above is a way to represent a bend down in the vertical plane, it could be defined more simply by
VBEND: SBEND, L=6.0, ANGLE=0.01,TILT


Figure 3.5: Reference System for a Rotation Around the s-Axis

### 3.16 BEAMBEAM Element

```
label: BEAMBEAM, TYPE=name,SIGX=real,SIGY=real,XMA=real,YMA=real,&
    CHARGE=real
```

The command BEAMBEAM may be inserted in a beam line to simulate a beam-beam interaction point. ${ }^{1}$ It has five real attributes:

SIGX The horizontal extent (standard deviation) of the opposite beam (default: 0 m ).
SIGY The vertical extent (standard deviation) of the opposite beam (default: 0 m ).
XMA The horizontal displacement of the opposite beam with respect to the ideal orbit (default: 0 m ).

YMA The vertical displacement of the opposite beam with respect to the ideal orbit (default: 0 m ).

CHARGE The charge of particles in the opposite beam in proton charges (default: 0). Entering the keyword CHARGE alone sets the charge to -1 .

[^0]A beam-beam interaction requires the particle energy (ENERGY) and the charge of the particles in the beam considered (CHARGE), as well as the number of particles per bunch (NPART) to be set by a BEAM command (see Section 5.2) before any calculations are performed. Example:

```
BEAM, PARTICLE=POSITRON,NPART=1.E12,ENERGY=50.0
BB: BEAMBEAM,SIGX=1.E-3,SIGY=5.E-4,CHARGE=1.
```


### 3.17 Arbitrary MATRIX Element

```
MATRIX, TYPE=name,RM (1,1)=real, .., RM (6,6)=real,&
    TM (1, 1, 1) =real, . . , TM (6,6,6)=real
```

The MATRIX permits definition of an arbitrary transfer matrix. It has two real array attributes:
RM ( $\mathrm{i}, \mathrm{k}$ ) Defines the element $(i, k)$ of the linear transfer matrix.
TM ( $i, k, 1$ ) Defines the element ( $i, k, l$ ) of the second-order TRANSPORT matrix.
Matrix elements not entered are taken from the identity transformation, i.e.

$$
\operatorname{RM}(\mathrm{i}, \mathrm{k})=\left\{\begin{array}{l}
1, \text { if } i=k, \\
0, \text { if ifk, }
\end{array} \quad \operatorname{TM}(\mathrm{i}, \mathrm{k}, \mathrm{l})=0 .\right.
$$

### 3.18 Editing Element Definitions

The EDIT command of former MAD versions is no longer available, but an element definition can be changed in two ways:

- Entering a new definition: The element will be replaced in the main beam line expansion.
- Entering the element name together with new attributes: The element will be updated in place, and the new attribute values will replace the old ones.

This example shows two ways to change the strength of a quadrupole:

```
QF: QUADRUPOLE,L=1,K1=0.01 ! Original definition of QF
QF: QUADRUPOLE,L=1,K1=0.02 ! Replace whole definition of QF
QF,K1=0.02 ! Replace value of K1
```

When the type of the element remains the same, replacement of an attribute is the more efficient way.
Element definitions can be edited freely. The changes do not affect already defined objects which belong to its class (see Section 3.19). The element to be modified should not be member of a beam line sequence (see Section 4.2).

Beam elements or beam lines (see Section 4) can be replaced by other beam elements or beam lines. However, if a beam line is involved in the replacement, and the replaced item occurs in the working beam line (see Section 5.1), the working beam line becomes obsolete and is deleted automatically.

### 3.19 Element Classes

The concept of element classes solves the problem of addressing instances of elements in the accelerator in a convenient manner. It will first be explained by an example. All the quadrupoles in the accelerator form a class QUADRUPOLE. Let us define three subclasses for the focussing quadrupoles, the defocussing quadrupoles, and the skewed quadrupoles:

```
MQF: QUADRUPOLE,L=LQM,K1=KQD ! Focussing quadrupoles
MQD: QUADRUPOLE,L=LQM,K1=KQF ! Defocussing quadrupoles
MQT: QUADRUPOLE,L=LQT,TILT ! Skewed quadrupoles
```

These classes can be thought of as new keywords which define elements with specified default attributes. We now use theses classes to define the real quadrupoles:

```
QD1: MQD ! Defocussing quadrupoles
QD2: MQD
QD3: MQD
QF1: MQF ! Focussing quadrupoles
QF2: MQF
QF3: MQF
QT1: MQT,K1=KQT1 ! Skewed quadrupoles
QT2: MQT,K1=KQT2
```

These quadrupoles inherit all unspecified attributes from their class. This allows to build up a hierarchy of objects with a rather economic input structure.

The full power of the class concept is revealed when object classes are used to select instances of elements for printing. Example:

```
PRINT,QUADRUPOLE ! Print at all quadrupoles
PRINT,MQT[1/2] ! Print at the first two skewed quadrupoles
```

More examples appear in Section 5.3.
More formally, for each element keyword MAD maintains a class of elements with the same name. A defined element becomes itself a class which can be used to define new objects, which will become members of this class. A new object inherits all attributes from its class; but its definition may override some of those values by new ones. All class and object names can be used in range selections, providing a powerful mechanism to specify positions for matching constraints and printing.

### 3.20 LUMP Element

The command LUMP concatenates transfer maps. It has the form

```
label: LUMP,TYPE=name,ORDER=integer,LINE=line
```

It expands the beam line line, and evaluates its transfer map to order ORDER. The map is associated with the name label. In other words this command pre-calculates a transfer map and stores it for future use.

The ORDER parameter must lie in the range $2 \leq$ ORDER $\leq 6$. It refers to the order of the corresponding Hamiltonian; ORDER $=2$ denotes a linear transformation, ORDER $=3$ includes all terms up to second order in the TRANSPORT sense.

In a TWISS command MAD considers terms up to ORDER $=3$. To handle chromatic effects properly, all LUMPs used in a beam line should have ORDER $\geq 3$. However, in a TWISS command MAD ignores any terms with ORDER $>3$. MAD is not capable to find the integral part of the phase advance over a LUMP. Thus, for ORDER $\geq 2 \alpha, \beta, D$ are correct, as well as the fractional parts of $\mu$. If ORDER $\geq 3$, the chromatic functions are also correct.

For tracking by the TRANSPORT method MAD limits the order of tracking to three. For tracking bye the Lie-algebraic method, it always uses the order of the LUMP definition. For linear elements, tracking can be speeded up considerably, if one selects ORDER $=2$.

Some important points must be kept in mind about this element:

- The transfer map is evaluated when used for the first time, it is only re-evaluated when parameters of the line change.
- Any sub-lines must be defined before using the LUMP. MAD may otherwise complain about undefined sub-lines when tracking.
- The name label behaves like an element, it is not a beam line. The individual elements building up the LUMP cannot be misaligned nor mispowered. It is however permissible to misalign (not to mispower) the LUMP as a whole. A LUMP can also be used in a SURVEY command. It must not be used in a HARMON run.
- Concatenation of too many elements causes truncation of high-order effects. The number of concatenated elements should therefore not be too large.
- As from Version 8.2/4 of MAD transfer maps are recalculated automatically for a lump if one of the elements contained in this lump is changed. Use of a lump in matching is always safe. However lumping saves time only if the elements in a lump are not varied.

Examples for LUMP definitions:
SEC: LINE=(QF,D,QD,D)
LSEC: LUMP, ORDER=4,LINE=SEC
Putting the line sequence directly in the LUMP has the same effect:
LSEC: LUMP, ORDER=4, LINE=(QF,D, QD, D)
In earlier versions of MAD it could save time when bending magnets were defined as lumps. From Version 8 MAD treats bending magnets as lumps automatically, it will be useless if the user attempts to do so.

# Chapter 4. Beam Lines, Sequences, Lists, Ranges, Selections 

Table 4.1: Line Definition Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| LINE | Beam line definition | 4.1 |
| SEQUENCE | Beam line sequence | 4.2 |
| LIST | Replacement list definition | 4.6 |

### 4.1 Beam Lines

The accelerator to be studied is known to MAD as a sequence of physical elements called a beam line. A beam line is built from simpler beam lines whose definitions can be nested to any level. A powerful syntax allows to repeat, to reflect, or to replace pieces of beam lines. Formally a beam line is defined by a LINE command:

```
label(arg{,arg}): LINE=(member{,member})
```

Label gives a name to the beam line for later reference. The formal argument list (arg\{,arg\}) is optional (see below). Each member may be one of the following:

- Element label,
- Beam line label,
- Sub-line, enclosed in parentheses,
- Formal argument name,
- Replacement list label.

Beam lines may be nested to any level.

### 4.1.1 Simple Beam Lines

The simplest beam line consists of single elements:

```
label: LINE=(member{,member})
```

Example:

$$
\begin{array}{ll}
\mathrm{L}: & \operatorname{LINE}=(A, B, C, D, A, D) \\
& \text { USE, } \mathrm{L}
\end{array}
$$

The USE command tells MAD to perform all subsequent calculations on the sequence

$$
A, B, C, D, A, D
$$

### 4.1.2 Sub-lines

Instead of referring to an element, a beam line member can refer to another beam line defined in a separate command. This provides a shorthand notation for sub-lines which occur several times in a beam line. Lines and sub-lines can be entered in any order, but when a line is expanded, all its sub-lines must be known. Example:

```
L: LINE=(A,B,S,B,A,S,A,B)
S: LINE=(C,D,E)
USE,L
```

This example produces the following expansion steps:

1. Replace sub-line $S$ :

$$
(A, B,(C, D, E), B, A,(C, D, E), A, B)
$$

2. Omit parentheses:

$$
A, B, C, D, E, B, A, C, D, E, A, B
$$

### 4.1.3 Reflection and Repetition

An unsigned repetition count and an asterisk indicate repetition of a beam line member. A minus prefix causes reflection, i.e. all elements in the subsequence are taken in reverse order. Sub-lines of reflected lines are also reflected, but physical elements are not. If both reflection and repetition are desired, the minus sign must precede the repetition count. Example:

```
R: LINE=(G,H)
S: LINE=(C,R,D)
T: LINE=(2*S,2*(E,F),-S,-(A,B))
    USE,T
```

Proceeding step by step, this example produces

1. Replace sub-line $S:$

$$
((C, R, D),(C, R, D),(E, F),(E, F),(D,-R, C),(B, A))
$$

2. Replace sub-line R:

$$
((C,(G, H), D),(C,(G, H), D),(E, F),(E, F),(D,(H, G), C),(B, A))
$$

3. Omit parentheses:

$$
C, G, H, D, C, G, H, D, E, F, E, F, D, H, G, C, B, A
$$

Note that the inner sub-line $R$ is reflected together with the outer sub-line $S$.

### 4.1.4 Replaceable Arguments

A beam line definition may contain a formal argument list, consisting of labels separated by commas and enclosed in parentheses. Such a line can be expanded for different values of its arguments. When it is referred to, its label must be followed by a list of actual arguments separated by commas and enclosed in parentheses. Each top level member of the list represents one argument; it can contain anything which may occur as a beam line member. MAD assumes an implicit pair of parentheses around each argument. The number of actual arguments must agree with the number of formal arguments. All occurrences of a formal argument on the right-hand side of the line definition are replaced by the corresponding actual argument. Example:

```
S: LINE=(A,B,C)
L(X,Y): LINE=(D,X,E,3*Y)
    USE, L( }4*F,-2*S
```

Proceeding step by step, this example generates the expansion

1. Replace formal arguments:

$$
(\mathrm{D},(4 * \mathrm{~F}), \mathrm{E}, 3 *(-2 *(\mathrm{~A}, \mathrm{~B}, \mathrm{C})))
$$

2. Combine repetition counts for last sub-line:

$$
(D,(4 * F), E,-6 *(A, B, C))
$$

3. Expand repetitions and reflection:

$$
(D,(F, F, F, F), E,(C, B, A),(C, B, A),(C, B, A),(C, B, A),(C, B, A),(C, B, A))
$$

4. Omit parentheses:

$$
D, F, F, F, F, E, C, B, A, C, B, A, C, B, A, C, B, A, C, B, A, C, B, A
$$

Second example:

```
CEL(SF,SD): LINE=(QF,D,SF,D,B,D,QD,D,SD,D,B,D)
ARC: LINE=(CEL(SF1,SD1),CEL(SF2,SD2),CEL(SF1,SD1))
    USE,ARC
```

This example generates the expansion

1. Replace the line CEL and its formal arguments:
```
( (QF, D, (SF1) , D, B, D, QD, D, (SD1) , D,B,D)
(QF,D, (SF2) , D, B, D, QD, D, (SD2) , D, B, D)
(QF, D, (SF1) , D, B, D, QD, D, (SD1), D,B,D))
```

2. Omit parentheses:
```
QF,D,SF1,D,B,D,QD,D,SD1,D,B,D
QF,D,SF2,D,B,D,QD,D,SD2,D,B,D
QF,D,SF1,D,B,D,QD,D,SD1,D,B,D
```


### 4.2 Beam Line Sequences

When MAD runs in an accelerator control system, the sequence of elements is usually generated from a data base. For this purpose there is a command

```
label: SEQUENCE,REFER=keyword
    object: class,AT=real{,attributes}
    ...
ENDSEQUENCE
```

It reads a sequence of element definitions, compiles a data bank which resembles a beam line definition, and gives it the name label. The resulting sequence can be used like a beam line. For each nondrift element in the sequence one element definition appears following the SEQUENCE command and preceding the ENDSEQUENCE command. These look like the definitions described in Chapter 3, with the following differences:

- The name object must be unique; it must not be defined earlier in the data.
- The second name class on the definition must be a class name.
- The class name must be followed immediately by the attribute AT=real. This specifies the longitudinal position of the element with respect to the beginning of the sequence. The attribute REFER specifies the reference points for the elements:

CENTRE The element centre is placed at the given position (default).
ENTRY The element entry is placed at the given position.
EXIT The element exit is placed at the given position.
The elements must be entered in order of increasing position, and must not overlap.

- The elements object inherit the attributes from class. If no further attributes appear, MAD uses the data bank of class for the element data. In this case class cannot be modified subsequently. If further attributes exist, MAD makes a copy of the data for the class, as if it had read a normal element definition.

When the sequence is expanded in a USE command, MAD generates the drift spaces for proper positioning.

For efficiency reasons MAD imposes an important restriction for variable lengths. If the lengths of elements and their positions are defined as constant values, MAD gives the intervening drifts a constant length as well, and checks that the drift lengths are all positive. In this case the drift lengths can never be varied, and a change in element length affects the total length of the system.

Variation of an element position and/or length is possible. The element length or position to be varied must then be defined by an expression. This causes MAD to adjust the dependent drift length(s) such as to keep all definitions consistent. However, it is the responsibility of the program user that drift spaces do not become negative. Example:

```
! Define element classes for a simple cell:
B: SBEND,L=35.09, ANGLE = 0.011306116
QF: QUADRUPOLE,L=1.6,K1=-0.02268553
QD: QUADRUPOLE,L=1.6,K1=0.022683642
SF: SEXTUPOLE,L=0.4,K2=-0.13129
SD: SEXTUPOLE,L=0.76,K2=0.26328
! Define the cell as a sequence:
CELL: SEQUENCE
    B1: B, AT=19.115
    SF1: SF, AT=37.42
    QF1: QF, AT=38.70
    B2: B, AT=58.255,ANGLE=B1[ANGLE]
    SD1: SD, AT=76.74
    QD1: QD, AT=78.20
    ENDM: MARKER, AT=79.0
ENDSEQUENCE
```

In this example all members of the sequence use the storage of their class except the element B 2 which uses a copy because one of its attribute has been redefined.

### 4.3 Editing a Beam Line Sequence

A beam line sequence may be edited before it is used in a USE command. Seven commands are available for this purpose:

```
SEQEDIT,SEQUENCE=name
INSTALL,ELEMENT=name,AT=value [,FROM=name]
REMOVE,ELEMENT=name
MOVE,ELEMENT=name [,BY=value] [,TO=value [,FROM=name]]
REFLECT
CYCLE,START=name
ENDEDIT
```

They are explained in detail in the next sub-sections.

### 4.3.1 Making a sequence known for editing

Before modifying a sequence, its name must be set for editing by the command

```
SEQEDIT,SEQUENCE=name
```

MAD will make the sequence name the current sequence being edited. Once editing is finished, one should give the command

```
ENDEDIT
```


### 4.3.2 Installing an Element

A new element can be installed in the edited sequence by the command
INSTALL, ELEMENT=name, AT=value [,FROM=name]

It has the following attributes:
ELEMENT The name of the element to be installed.
at The position where to install the element.
FROM The name of the element to be taken as the origin. If the FROM attribute does not appear, AT implies an absolute displacement from the beginning of the sequence; if it does, the distance is relative to the position of that element. A relative position may be negative.

### 4.3.3 Removing Elements

The command

```
REMOVE,ELEMENT=name,PATTERN=string
```

removes all elements from the sequence which match at least one of the two following conditions:

- The element is member of the class name,
- The element name matches the "wild-card" pattern string.
- wild-card

Examples:
REMOVE, ELEMENT=DRIFT
REMOVE,PATTERN="S.*QD.*R1"
The first command removes all explicit drifts from the sequence, and the second command removes the elements beginning with the letter S, containing the string QD, and ending with the string .R1. Note that the two occurrences of .* each stand for an arbitrary number (including zero) of any character, and the occurrence $\backslash$. stands for a literal period. For definitions of patterns refer to Section 1.7.

### 4.3.4 Moving an Element

The command

```
MOVE,ELEMENT=name [,BY=value] [,TO=value [,FROM=name]]
```

moves an element to a new location. Its attributes are:
ELEMENT The name of the element to be moved.
BY The amount by which the element is to be moved.

T0
FROM

The new position for the element.
The name of the element to be taken as the origin. If the FROM attribute does not appear, TO implies an absolute displacement from the beginning of the sequence; if it does, the distance is relative to the position of that element. A relative position may be negative. The MOVE command may change the order of elements in the sequence.

### 4.3.5 Reflecting a Sequence

The command
REFLECT
reverses the order of all elements in the sequence currently being edited. All further edit commands must refer to the new positions.

### 4.3.6 Cyclic interchange

The command

```
CYCLE, START=name
```

makes a cyclic interchange of all elements in the current edit sequence so as to start at the element name. This element should preferably be a MARKER. All further edit commands must refer to the new positions.

### 4.3.7 Example for the Sequence Editor

```
SEQ:SEQUENCE
    B1: B, AT=19.115
    SF1: SF, AT=37.42
    QF1: QF, AT=38.70
    B2: B, AT=58.255
    SD1: SD, AT=76.74
    QD1: QD, AT=78.20
    ENDM: MARKER, AT=79.0
ENDSEQ
B2W:B2,ANGLE=0.1*B2[ANGLE]
SEQEDIT,SEQUENCE=SEQ
MOVE, ELEMENT=SF1,TO=-1.27,FROM=QF1
MOVE, ELEMENT=SD1, BY=0.01
REMOVE,ELEMENT=B2
INSTALL,ELEMENT=B2W,AT=58.255
ENDEDIT
```

This example moves the two sextupoles and replaces the element B2 by the element B2W.

### 4.4 More Examples for Beam Lines

### 4.4.1 CERN SPS Lattice

The CERN SPS lattice may be represented using the following beam elements:

```
QF: QUADRUPOLE,... ! focussing quadrupole
QD: QUADRUPOLE,... ! defocussing quadrupole
B1: RBEND,... ! bending magnet of type 1
B2: RBEND,... ! bending magnet of type 2
DS: DRIFT,... ! short drift space
DM: DRIFT,... ! drift space replacing two bends
DL: DRIFT,... ! long drift space
```

The SPS machine is represented by the lines

```
SPS: LINE=(6*SUPER)
SUPER: LINE=(7*P44,INSERT,7*P44)
INSERT: LINE=(P24,2*P00,P42)
POO: LINE=(QF,DL,QD,DL)
P24: LINE=(QF,DM,2*B2,DS,PD)
P42: LINE=(PF,QD,2*B2,DM,DS)
P44: LINE=(PF,PD)
PD: LINE=(QD,2*B2,2*B1,DS)
PF: LINE=(QF,2*B1,2*B2,DS)
```

In order not to overload the example, small gaps between magnetic elements have been omitted.

### 4.4.2 LEP Lattice

A similar method of defining accelerator structures has been used in Reference [16] to define the LEP structure. Translation of those element sequences to the MAD input format gives:

```
LEP: LINE=(4*SUP)
SUP: LINE=(OCT,-OCT)
OCT: LINE=(LOBS,RFS,DISS,ARC,DISL,RFL,LOBL)
LOBS: LINE=(L1,QS1,L2,QS2,L3,QS3,L4,QS4)
RFS: LINE=(L5,QS5,L5,QS6,L5,2*(QS7,L5,QS8,L5))
DISS: LINE=(QS11,L25,BW,L22,QS12,L25,B4,L22,QS13,L25,B4,
        L22,QS14,L25,B4,L31, QS15,L25,B4,L32,SF,L23,QS16)
ARC: LINE=(L21,B6,L22,SD,L23,QD,
        7*(CELL(SF1,SD1),CELL(SF,SD)),CELL(SF1,SD1),
        L24,B6,L41,QF,L21,B6,L22,SD4,L23,QD,
        7*(CELL(SF4,SD3),CELL(SF3,SD4)),CELL(SF4,SD3),
        L24,B6,L22,SF3,L23)
DISL: LINE=(QL16,L34,B4,L22,QL15,L33,B4,L22,QL14,L25,B4,
            L22, QL13, L25, B4, L22, QL12, L25,BW, L22 , QL11)
RFL: LINE=(2*(L5,QL8,L5,QL7),L5,QL6,L5,QL5,L5)
LOBL: LINE=(QL4,L14,QL3,L13,QL2,L12,QL1,L11)
BW: LINE=(W,L26,W)
B4: LINE=(B,L26,B)
```

```
B6: LINE=(B,L26,B,L26,B)
CELL(SF,SD): LINE=(L24,B6,L22,SF,L23,QF,L21,B6,L22,SD,L23,QD)
```

Here the element definitions have been left open.

### 4.5 Beam Line References

Many MAD commands require a reference to a beam line. Normally one enters the label of a beam line. If the beam line is defined with formal arguments, the label is followed by an actual argument list. MAD also accepts a sequence enclosed by parentheses instead of a name. Examples:

```
USE,CELL1
USE, CELL(SF1,SD1)
USE,(QF,D,QD,D)
```


### 4.6 Replacement Lists

The replacement list provides a powerful tool to describe complex machines. It replaces a set of identical labels in a sequence in turn by different items. A successive refinement of the machine description is made possible by redefining elements as beam lines or as replacement lists. A replacement list definition resembles a beam line definition:

$$
\text { label: } \operatorname{LIST}=(\text { member }\{, \text { member }\})
$$

All items which can occur in a beam line can also occur in a list. The effect of a replacement list on a beam line is defined as follows:

1. Expand all replacement lists to the level of single labels. If a list member is a beam line name, do not expand it. Mark the first label of each expanded list as current.
2. Expand the given beam line. Each time the label of a replacement list is encountered, replace it by the current label in this list and mark the following label of the list as current.
3. If a substituted label is a beam line, expand it.

To study several arrangements of sextupole families, one may define one or more replacement lists for each arrangement. There is no need to change anything else in the machine structure. Example:

```
RING: LINE=(5*(QF,D,SF,D,B,D,QD,D,SD,D,B))
SF: LIST=(2*(SF1,SF2),SF1)
SD: LIST=(2*(SD1,SD2),SD1)
    USE,RING
```

In this example MAD proceeds as follows:

1. Expand the two lists:
```
SF: LIST=(SF1,SF2,SF1,SF2,SF1)
SD: LIST=(SD1,SD2,SD1,SD2,SD1)
```

2. Expand the line RING:

$$
\text { RING: } \quad \begin{aligned}
\text { LINE }= & (Q F, D, S F, D, B, D, Q D, D, S D, D, B, D, \\
& Q F, D, S F, D, B, D, Q D, D, S D, D, B, D, \\
& Q F, D, S F, D, B, D, Q D, D, S D, D, B, D, \\
& Q F, D, S F, D, B, D, Q D, D, S D, D, B, D, \\
& Q F, D, S F, D, B, D, Q D, D, S D, D, B, D)
\end{aligned}
$$

3. Replace labels in RIvG:

$$
\text { RING: } \begin{aligned}
\text { LINE }= & (Q F, D, S F 1, D, B, D, Q D, D, S D 1, D, B, D, \\
& Q F, D, S F 2, D, B, D, Q D, D, S D 2, D, B, D, \\
& Q F, D, S F 1, D, B, D, Q D, D, S D 1, D, B, D, \\
& Q F, D, S F 2, D, B, D, Q D, D, S D 2, D, B, D, \\
& Q F, D, S F 1, D, B, D, Q D, D, S D 1, D, B, D)
\end{aligned}
$$

In a second example, we replace some drift spaces by special insertions:

```
FODO: LINE=(4*(QF,D,QD,D))
D: LIST=(DD,DC,3*DD,DC,2*DD)
DD: DRIFT,L=2.0
DC: LINE=(D1,PICKUP,D1)
D1: DRIFT,L=0.5
PICKUP: MONITOR,L=1.0
    USE, FODO
```

In this example we have the following steps:

1. Expand the list D (note that DC is not expanded yet):

$$
\text { D: } \quad \text { LIST=(DD }, D C, D D, D D, D D, D C, D D, D D)
$$

2. Expand the beam line FODO:

$$
\text { FODO: } \quad L I N E=(Q F, D, Q D, D, Q F, D, Q D, D, Q F, D, Q D, D, Q F, D, Q D, D)
$$

3. Replace labels in the beam line FODO:
```
FODO: LINE=(QF,DD,QD,DC,QF,DD,QD,DD,
        QF,DD,QD,DC,QD,DD,QD,DD)
```

4. Expand the line DC:
```
FODO: LINE=(QF,DD,QD,D1,PICKUP,D1,QF,DD,QD,DD,
            QF,DD,QD,D1,PICKUP,D1,QF,DD,QD,DD)
```

Replacement lists are circular. When a list is exhausted, and more replacement labels are required, the current element is reset to the first one, and replacement continues. Without changing the result, the lists in the first example could therefore also be written as

```
SF: LIST=(SF1,SF2)
SD: LIST=(SD1,SD2)
```

Replacement lists should not be used when lines are reflected. If a replacement list occurs within a reflected part of a beam line, MAD never reflects the replacement list and gives a warning message. Unless the list happens to be symmetric, the expansion would be wrong. Care should also be taken when expanding sub-lines of lines which refer to replacement lists. By definition, MAD always begins replacement at the first label of a replacement list. Here is an example what could go wrong:

```
CELL1: LINE=(...,SF,...)
CELL2: LINE=(...,SF,...)
CELL3: LINE=(...,SF,...)
SF: LIST=(SF1,SF2,SF3)
ARC: LINE=(CELL1,CELL2,CELL3)
    USE, CELL3
```

After the command USE, ARC the cell CELL3 will contain SF3. However, after the command USE, CELL3 the cell CELL3 will contain SF1. In case of doubt, use the PRINT command and check the correct replacement.

### 4.7 Ranges and Sets of Observation Points in a Line

Many MAD commands allow for the possibility to process or display a range or a subset of the elements occurring in the beam line expanded by the latest USE command. There are two selection mechanisms:

- A range (an interval) can be defined starting at the exit of a given element and ending at the exit of another element. The syntax is

```
element-occurrence 1 / element-occurrence 2
```

If the range consists of only one element or, it may be entered as

```
element-occurrence
```

An element-occurrence can be defined by

- A MAD index:
\#index
index is the number of physical elements preceding the position in the expanded beam line. There are four predefined MAD indices:
\#S The start of the full beam line expanded by USE,
\#E The end of the full beam line expanded by USE,
\#F The exit of the first element in RANGE as read by USE,
\#L The exit of the last element in RANGE as read by USE.
- A class or element name and its occurrence count:


## label [occurrence]

where label is a class or element name, and occurrence is the occurrence number of that name in the expanded beam line.

- The explicit element name, if there is only one occurrence.

```
label
```

Names are always unique in a SEQUENCE.

- A set (a selection) of elements belonging to the same class (e. g. QUADRUPOLE) or beam line is made by

```
label[occurrence 1/occurrence 2]
```

If label is a class name, this refers to all elements of that class, starting at a occurrence 1 and ending at occurrence 2 . If label is a beam line name, this refers to all elements belonging to that beam line, starting at a occurrence 1 and ending at occurrence 2. Each occurrence is an occurrence number for the class or beam line. A single occurrence is selected by the syntax

```
label[occurrence]
```

If all occurrences are implied, one may simply use

- A range

Occurrences and physical elements are numbered relative to the entire beam line expanded by the latest USE command. They are not affected when RANGE is set in the USE command.

### 4.7.1 Examples

Assume the following definitions:

```
M: MARKER
S: LINE=(C,M,D)
L: LINE=(A,M,B,2*S,A,M,B)
    USE ,L , RANGE=M[2]/M [3]
```

The line $L$ is equivalent to the sequence

$$
\mathrm{A}, \mathrm{M}, \mathrm{~B}, \mathrm{C}, \mathrm{M}, \mathrm{D}, \mathrm{C}, \mathrm{M}, \mathrm{D}, \mathrm{~A}, \mathrm{M}, \mathrm{~B}
$$

Examples for ranges:
\#4/\#6 C[1],M[2],D[1],
S[1] The entire first sub-line $S$, that is from entry of first $S$ to exit of first $S$, both included,
S
All positions within both occurrences of S , including entries and exits of S ,
A[1]/A[2] Positions A[1] to A[2] including all intermediate positions,
M[2]/M[3] From second to third M, also known as \#F/\#L,
L The entire line L, can also be written as \#S/\#E.
Examples for single positions:
\#O or \#S At the beginning of the line L,
C[1] After the first element C, can be written as \#4,
M[3] After the third marker M, also \#8 or \#L,
\#E After the last element of the line L.
Examples for multiple positions:
A
All A[i], that is A[1] and A[2],
A[1/2] A[1] and A[2] only,
M
$M[2 / 3] \quad M[2]$ and $M[3]$ only.
More examples can be found in Section 5.10.

## Chapter 5. Action Commands

Table 5.1: Environment Setting and Action Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| USE | Select working beam line | 5.1 |
| BEAM | Beam data: particle energy and charge, emittances etc. | 5.2 |
| PRINT | Select print positions | 5.3 |
| SELECT | Select output positions | 5.3 |
| SURVEY | Print geometry of machine | 5.5 |
| TWISS | Print lattice functions | 5.6 |
| IBS | Intra-beam scattering | 5.7 |
| EMIT | Equilibrium emittances | 5.8 .1 |
| EIGEN | Eigenvectors for normal modes | 5.8 .2 |
| ENVELOPE | Beam envelopes in 3 degrees of freedom | 5.8 .2 |
| TWISS3 | Mais-Ripken lattice functions | 5.8 .2 |
| DYNAMIC | Dynamic normal form analysis | 5.9 |
| STATIC | Static normal form analysis | 5.9 |
| SPLIT | Interpolate within element for 0PTICS command | 5.4 |
| OPTICS | Output lattice functions and element strengths for control | 5.10 |
|  | system |  |

### 5.1 USE Statement

The USE statement specifies the beam line and its range to be used in subsequent commands. It must be entered before any physics computation is requested. It has the form
USE , PERIOD=line, RANGE=range, SYMM, SUPER=integer

If the PERIOD attribute appears, the USE command causes the following actions:

- Replace formal arguments by the corresponding actual arguments,
- Insert sub-lines in the proper places,
- Expand reflections and repetitions of sub-lines,
- Reset all selection flags (see Section 5.3),
- Clear all error definitions (see Chapter 8),

A data structure is generated containing one entry for each beam element occurring in the beam line, and one entry for the entrance and exit of each beam line or sub-line. This structure remains in memory until another USE creates a new one, or until the replacement of a beam element or beam line makes it invalid. A subsequent USE command referring to the same beam line will recreate the structure, and effectively clear all selection flags and error definitions.

If the RANGE attribute appears, USE marks beginning and end of range for future use. All subsequent commands which require a beam line operate on the range selected. Example:

```
A: LINE=(...,B,\ldots) ! definition of A
B: LINE=(...) ! definition of B
    USE,A! ! expand beam line A
```

```
B: LINE=(...)
```

```
some actions
```

some actions
redefinition of B kills sequence
redefinition of B kills sequence
new expansion of beam line A
new expansion of beam line A
more actions

```
more actions
```

The USE statement has four attributes:
PERIOD The beam line to be expanded (see Section 4.5). If omitted, the previous line is used without a new expansion.

RANGE The range of the beam line to be used. If PERIOD is given and RANGE is omitted, the range is the complete line. If PERIOD and RANGE are both omitted, the previous line and range are assumed. If RANGE is given, but PERIOD omitted, a new range is selected from the previous line.

SYMM A logical flag. If set, subsequent calculations are made as if the mirror image had been appended to the range.

SUPER An integer. Specifies the number of superperiods desired in the calculations. Quantities like tunes, chromaticities, and the like which refer to the machine circumference will be scaled with the value of SUPER (default: 1 ).

Example:

```
OCT: LINE=(...) ! one octant of the machine
USE , OCT , SYMM , SUPER=4
```

Here the beam line OCT is expanded. The effect is the same as if we had expanded the full machine as follows:

TURN: LINE=(4*(OCT,-0CT))
However, MAD needs only to walk through one octant, instead of through the whole machine. If the beam line is defined with a formal argument list, an actual argument list must be provided:

```
CELL(SF,SD): LINE=(...)
    USE,PERIOD=CELL(SF1,SD1)
```

In this example, one super-period is used. The beam line CELL is expanded with the formal arguments $S F$ and $S D$ replaced by $S F 1$ and $S D 1$ respectively. It is neither repeated nor made symmetric, since both SUPER and SYMM have been omitted.

### 5.2 BEAM Command

Many commands in MAD require the setting of various quantities related to the beam in the machine. These are entered by a BEAM command:

```
BEAM, PARTICLE=name,MASS=real,CHARGE=real,&
    ENERGY=real,PC=real,GAMMA=real,&
    EX=real,EXN=real,EY=real,EYN=real,&
    ET=real,SIGT=real,SIGE=real,&
    KBUNCH=integer,NPART=real,BCURRENT=real,&
    BUNCHED=logical,RADIATE=logical
```

Warning: BEAM updates, i. e. it replaces attributes explicitely mentioned, but does not return to default values for others! To reset to defaults, use RESBEAM (see below). The particle mass and charge are defined by:

PARTICLE The name of particles in the machine. MAD knows the mass and the charge for the following particles:

POSITRON The particles are positrons (default MASS $=m_{e}$, CHARGE $=1$ ),
ELECTRON The particles are electrons (MASS $=m_{e}$, CHARGE $=-1$ ),
PROTON The particles are protons ( $\mathrm{MASS}=m_{p}, \mathrm{CHARGE}=1$ ),
ANTI-PROTON The particles are anti-protons (MASS $=m_{p}$, CHARGE $=-1$ ).
For other particle names one may enter:
MASS The particle mass in GeV .
CHARGE The particle charge expressed in elementary charges.
By default the total particle energy is 1 GeV . A different value can be defined by one of the following:
ENERGY The total energy per particle in GeV . If given, it must be greater then the particle mass.

PC The momentum per particle in $\mathrm{GeV} / \mathrm{c}$. If given, it must be greater than zero.
GAMMA The ratio between total energy and rest energy of the particles $\gamma=E / m_{0}$. If given, it must be greater than one. If the mass is changed a new value for the energy should be entered. Otherwise the energy remains unchanged, and the momentum and $\gamma$ are recalculated. The emittances are defined by:

EX The horizontal emittance $E_{x}=\sigma_{x}^{2} / \beta_{x}($ default: 1 m$)$.
EY $\quad$ The vertical emittance $E_{y}=\sigma_{y}^{2} / \beta_{y}$ (default: 1 m ).
ET The longitudinal emittance $E_{t}=\sigma_{e} /\left(p_{0} c\right) \cdot c \sigma_{t}$ (default: 1 m ). The emittances can be replaced by the normalised emittances and the energy spread:

EXN $\quad$ The normalised horizontal emittance [m]: $E_{x n}=4 \beta \gamma E_{x}$ (ignored if $E_{x}$ is given).
EYN $\quad$ The normalised vertical emittance [m]: $E_{y n}=4 \beta \gamma E_{y}$ (ignored if $E_{y}$ is given).
SIGT The bunch length $c \sigma_{t}[\mathrm{~m}]$.
SIGE The relative energy spread $\sigma_{e} / p_{0} c[1]$.
Certain commands compute the synchrotron tune $Q_{s}$ from the RF cavities. If $Q_{s} \neq 0$, the relative energy spread $\sigma_{e} / p_{0} c$ and the bunch length $c \sigma_{t}$ are

$$
\frac{\sigma_{e}}{p_{0} c}=\sqrt{\frac{2 \pi Q_{s} E_{t}}{\eta C}}, \quad c \sigma_{t}=\sqrt{\frac{\eta C E_{t}}{2 \pi Q_{s}}}
$$

where $C$ is the machine circumference, and $\eta=\left(1 / \gamma^{2}\right)-\left(1 / \gamma_{t r}^{2}\right)$. Finally, the BEAM command accepts
KBUNCH The number of particle bunches in the machine (default: 1 ).
NPPART The number of particles per bunch (default: 0 ).

BUNCHED A logical flag. If set, the beam is treated as bunched whenever this makes sense.
RADIATE A logical flag. If set, synchrotron radiation is considered in all bipolar magnets.
The BEAM command changes only the parameters entered. The command

```
RESBEAM
```

resets all beam data to their defaults (listed in Table 5.2). It is defined in the command dictionary as

```
RESBEAM:BEAM, PARTICLE="POSITRON",ENERGY=1.0,EX=1.0,EY=1.0,ET=1.0,&
    KBUNCH=1.0,NPART=0.0,BCURRENT=0.0,-BUNCHED, -RADIATE,-LIST
```

Table 5.2: Default Beam Data

| attribute | value | unit | attribute | value | unit |
| :--- | :---: | :--- | :--- | :---: | :--- |
| PARTICLE | POSITRON |  | ENERGY | 1 | GeV |
| EX | 1 | rad m | EY | 1 | rad m |
| ET | 1 | GeV s | KBUNCH | 1 | 1 |
| NPART | 0 | 1 | BCURRENT | 0 | A |
| BUNCHED | false |  | RADIATE | false |  |

Examples:

```
BEAM, PARTICLE=ELECTRON,ENERGY=50,EX=1.E-6,EY=1.E-8,SIGE=1.E-3
BEAM, RADIATE
RESBEAM
BEAM, EX=2.E-5,EY=3.E-7,SIGE=4.E-3
```

The first command selects electrons, and sets energy and emittances. The second one turns on synchrotron radiation. The last two select positrons (by default), set the energy to 1 GeV (default), clear the synchrotron radiation flag, and set the emittances to the values entered.

The BEAM command stores all specified attributes in a data bank with the name BEAM. Some program modules of MAD also store data into this bank. Real expressions may refer to data in the BEAM bank using the notation

```
BEAM[attribute-name]
```

This notation refers to the value of attribute-name found in the BEAM bank before the command containing the reference is executed. For example, after an EMIT command both emittances are stored in the BEAM bank, but the vertical emittance EY may become zero. For subsequent tracking one may set the new value of EY to $1 / 2$ times the old value of EX by entering

```
BEAM, EY=0.5*BEAM[EX]
```


### 5.3 PRINT and SELECT Statements

Each position in the beam line carries several associated selection flags. They are initially cleared by the USE command when the beam line is expanded. Output is selected by setting some of these flags by one of the commands

```
PRINT, RANGE=range{,range},TYPE=type{,type},FULL, CLEAR
SELECT, FLAG=name, RANGE=range{,range},TYPE=type{,type},FULL, CLEAR
```

Both commands have the same attributes to select positions:
RANGE With a single command up to five selection points or ranges can be entered. If more than five ranges are desired, several commands must be used. For definitions refer to Section 4.7.

TYPE With a single command up to five TYPE names can be entered. All elements carrying one of these type attributes will be selected. If more than five type names are desired, several commands must be used.

FULL A logical flag. If set, all flags of a specified type are set, and no other processing of attributes occurs.

CLEAR A logical flag. If set, all flags of a specified type are cleared before they are set in selected positions.

The limits for the number of ranges or types may be changed by editing the command dictionary. The PRINT command always affects the print flag for SURVEY (see Section 5.5), or TWISS (see Section 5.6). In SELECT the flag type is chosen by the attribute FLAG: Three of its possible values affect action commands:

TWISS A SELECT,TWISS statement is equivalent to PRINT. The two commands

```
PRINT,FULL
    SELECT,FLAG=TWISS,FULL
```

have identical effect.
OPTICS Selects output positions for OPTICS (see Section 5.10).
TRACK Selects print positions for tracking (see Chapter 13). Care must be taken in using this option, as it may generate a lot of output.

Four more values, intended for the programmer, specify debugging output:
FIRST Selects dumping of first-order transfer matrices for selected elements during closed orbit search in TWISS.

SECOND Selects dumping of second-order TRANSPORT maps for selected elements during their accumulation in TWISS.

REFER Selects dumping of first-order transfer matrices for selected elements during accumulation for adjusting RF cavities.

LIE Selects dumping of Lie-algebraic maps during their accumulation.
PRINT and/or SELECT command(s) must be placed after the USE command, and before any action command (e.g. TWISS) to be affected. Regardless of the setting of print flags, start and end points of the computation range are always printed by NORMAL, SURVEY, and TWISS. Examples:


More examples can be found near the end of this chapter.

### 5.4 SPLIT Command, Request Interpolation for OPTICS

The OPTICS command writes one table line for each element selected by SELECT, OPTICS. The output line contains the element parameters and the lattice functions for the element centre or its exit. The command
SPLIT, NAME=name, FRACTION=real, RANGE=range, range, TYPE=name, name, FULL, CLEAR
selects additional positions for output of the lattice functions only. The positions are given the name NAME, and element parameters in those positions are output as zero. The elements are selected as by SELECT, and FRACTION specifies a fraction of the length of the element where output is desired. Any number of points can be selected in the same element; output occurs in order of increasing $s$. Example:

```
SPLIT,NAME=B1,FRACTION=0.25,RANGE=B
SPLIT, NAME=B2, FRACTION=0.5,RANGE=B
SPLIT,NAME=B3,FRACTION=0.75,RANGE=B
```

Gives three lines for each $B$, at $1 / 4,1 / 2$, and $3 / 4$ of its length respectively and assigns the names $\mathrm{B} 1, \mathrm{~B} 2, \mathrm{~B} 3$ to the three positions.

### 5.5 SURVEY Statement

The SURVEY command computes the geometry of the machine:

```
SURVEY, X0=real,YO=real,ZO=real,&
    THETAO=real,PHIO=real,PSIO=real,TAPE=file-name
```

It operates on the working beam line entered in the latest USE command (see Section 5.1). Its parameter list specifies the initial position and orientation of the reference orbit in the global coordinate system ( $X, Y, Z$ ). Omitted attributes assume zero values. Valid attributes are:

| XO | The initial X coordinate $[\mathrm{m}]$. |
| :--- | :--- |
| YO | The initial Y coordinate $[\mathrm{m}]$. |
| ZO | The initial Z coordinate $[\mathrm{m}]$. |
| THETAO | The initial angle $\theta[\mathrm{rad}]$. |
| PHIO | The initial angle $\phi[\mathrm{rad}]$. |
| PSIO | The initial angle $\psi[\mathrm{rad}]$. |

TAPE
If TAPE=file-name appears MAD writes a full survey table on a disk file file-name. Appendix A describes the format of the file written. Appendix B explains the format for file names. TAPE alone is equivalent to TAPE="survey".

SURVEY prints one line for either end of the computation range and a summary. It also prints one line for each element and for the entrance and exit of each beam line, if this position has been selected by PRINT or by SELECT, FLAG=TWISS (see Section 5.3). The layout coordinates and angles have been defined in Section 1.3. Example:

SURVEY, TAPE=LAYOUT
This example computes the machine layout with zero initial conditions and writes the results on a file called Layout.

### 5.6 TWISS Statement

The TWISS command causes computation of the linear lattice functions, and optionally of the chromatic functions. It operates on the working beam line defined in the latest USE command (see Section 5.1). TWISS prints one or two lines for either end of the computation range and a summary. It also prints one or two lines for each element and for the entrance and exit of each beam line, if this position has been selected by PRINT or by SELECT, FLAG=TWISS (see Section 5.3). The variables used have been defined in Section 1.5. Three forms are distinguished for the TWISS command. In all three forms initial values for the phase angles MUX and MUY are accepted. The relative energy error DELTAP may be entered in one of the forms

```
DELTAP=real,real
DELTAP=initial:final:step
```

The first form lists several numbers, which may be general expressions, separated by commas. The second form specifies an initial value, a final value, and a step, which must be constant expressions, separated by colons. Mixtures of both forms are accepted. Examples:

```
DELTAP=0.001 ! a single value
DELTAP=0.001,0.005 ! two values
DELTAP=0.001:0.007:0.002 ! four values
DELTAP=0.001,0.005:0.03:0.005 ! seven values
```

If DELTAP is missing, MAD uses the value 0.0 . Further attributes common to all three forms of the TWISS statements are:

CHROM A logical flag. If set, MAD also computes the chromatic functions defined in [24].
COUPLE A logical flag. If set, MAD computes the coupled linear lattice functions as defined in $[14,26]$. In this case CHROM is ignored.

TAPE If TAPE=file-name appears MAD writes a full Twiss table on a disk file file-name. Appendix A describes the format of the file written. Appendix B explains the format for file names. TAPE alone is equivalent to TAPE="twiss".

SAVE If SAVE=table-name appears on the command, MAD creates a full Twiss table in memory and gives it the name table-name (an identifier). Entering SAVE alone is equivalent to SAVE=TWISS. This table includes linear lattice functions as well as the chromatic functions for all positions.

TUNES
If TUNES=table-name appears on the command, MAD creates a table of tunes and chromaticities versus the selected values of DELTAP and gives it the name table-name (an identifier). Entering TUNES alone is equivalent to TUNES=TUNES.

The tables are suited for plotting (see Chapter 10 ). They may be written on disk by the ARCHIVE command (see Section 7.5).

### 5.6.1 Twiss Parameters for a Period

The simplest form of the TWISS command is

```
TWISS, DELTAP=real{,value},CHROM,COUPLE,&
    TAPE=file-name,SAVE=table-name
```

It computes the periodic solution for the specified beam line for all values of DELTAP entered (or for DELTAP $=0$, if none is entered). Example:

```
USE , OCT, SYMM, SUPER=4
TWISS,DELTAP=0.001,CHROM,TAPE=0PTICS
```

This example computes the periodic solution for the linear lattice and chromatic functions for the beam line OCT, made symmetric and repeated in four superperiods. The DELTAP value used is 0.001 . Apart from saving computing time, it is equivalent to the command sequence

```
RING: LINE=(4*(OCT,-0CT))
    USE,RING
    TWISS,DELTAP=0.001,CHROM,TAPE=0PTICS
```


### 5.6.2 Initial Values for Twiss Parameters taken from a Periodic Line

It is often useful to track the lattice functions starting with the periodic solution for another beam line. If this is desired the TWISS command takes the form

```
TWISS, DELTAP=real{,value},LINE=beam-line,&
    MUX=real,MUY=real&
    TAPE=file-name,SAVE=table-name
```

No other attributes should appear in the command. For each value of DELTAP MAD first searches for the periodic solution for the beam line mentioned in LINE=beam-line. The result is used as an initial condition for the lattice function tracking. In this form of the TWISS command the SYMM flag and the value for SUPER are ignored. Example:

```
CELL: LINE=(...)
INSERT: LINE=(...)
    USE,INSERT
    TWISS,LINE=CELL,DELTAP=0.0:0.003:0.001,CHROM,TAPE
```

For four values of DELTAP the following happens: First MAD finds the periodic solution for the beam line CELL. Then it uses this solution as initial conditions for tracking the lattice functions of the beam line TWISS. The initial phases are not specified, they are set to zero. Output is also written on the file TWISS.

If any of the beam lines was defined with formal arguments, actual arguments must be provided:

```
CELL(SF,SD): LINE=(...)
INSERT(X): LINE=(...)
    USE,INSERT
    TWISS,LINE=CELL(SF1,SD1)
```


### 5.6.3 Twiss Parameters with Numerical Initial Values

Initial values for linear lattice functions may also be numerical. Two equivalent methods are available to specify initial values:

- In the TWISS command itself, as in

```
TWISS, BETX=real,ALFX=real,MUX=real,&
    BETY=real,ALFY=real,MUY=real,&
    DX=real,DPX=real,DY=real,DPY=real,&
    X=real,PX=real,Y=real,DPY=real,&
    WX=real,PHIX=real,DMUX=real,&
    WY=real,PHIY=real, DMUY=real,&
    DDX=real,DDY=real,DDPX=real,DDPY=real,&
    TAPE=file-name,SAVE=table-name,&
    DELTAP=real:real:real
```

- In a separate command, BETAO (see also Section 5.6.4):

```
INITIAL: BETAO,&
    BETX=real,ALFX=real,MUX=real,&
    BETY=real,ALFY=real,MUY=real,&
    DX=real,DPX=real,DY=real,DPY=real,&
    X=real,PX=real,Y=real, PY=real,T=real,PT=real,&
    WX=real,PHIX=real,DMUX=real,&
    WY=real, PHIY=real, DMUY=real,&
    DDX=real,DDY=real,DDPX=real,DDPY=real
TWISS, BETAO=INITIAL,TAPE=file-name,SAVE=table-name,&
    DELTAP=real:real:real
```

In the second case the TWISS command may override some of the values set by BETAO. All variables listed in Section 1.5.3 are permitted as input attributes, but BETX and BETY are required in either case. The flags SYMM and COUPLE, and the value for SUPER are ignored in this use of the TWISS command. As entered in this command, the initial conditions cannot depend on the quantity DELTAP, and can thus be correct only for one such value. This should be remembered when using this form of the TWISS command.

### 5.6.4 SAVEBETA Command, Save Lattice Parameters for Later Use

Sometimes it is useful to transfer computed lattice parameters to later commands. The command sequence

```
USE, . . .
SAVEBETA,LABEL=name, PLACE=place
TWISS,...
```

may help to do this. When reaching the place place during execution of TWISS MAD will create a BETAO bank with the name name. This bank is filled with the values of all lattice parameters in place. The SAVEBETA command has two attributes:

LABEL The name to be given to the created BETAO bank.
PLACE A position within the selected range of the working beam line.

Example 1:

```
USE, CELL
SAVEBETA,LABEL=END,PLACE=#E
TWISS
USE,INSERT
TWISS,BETAO=END
```

This will first calculate the periodic solution of the line CELL, and then track lattice parameters through INSERT, using all end conditions (including phases and orbit) in CELL to start.

Example 2:

```
USE, CELL
SAVEBETA,LABEL=END,PLACE=#E
TWISS
USE,INSERT
TWISS, BETX=END [BETY] , BETY=END [BETX]
```

This is similar to the first example, but the beta functions are interchanged (overwritten).

### 5.7 IBS Command, Intra-Beam Scattering

The IBS command takes an internal table (default: TWISS) generated by a previous TWISS,SAVE command and prints out information about intra-beam scattering. This command uses the method by Bjorken and Mtingwa [3, 9].

```
IBS,TABLE=table
```

This single attribute TABLE specifies the Twiss table to be used (default: TWISS). The IBS command uses the same print flags as TWISS to select output positions. Example:

```
TWISS,SAVE=LATTICE
IBS,TABLE=LATTICE
```


### 5.8 Emittances, Eigenvectors, Normal Modes, Beam Envelope

### 5.8.1 Electron Beam Parameters

The command
EMIT, DELTAP=real
adjusts the RF frequencies such as to obtain the specified average energy error. More precisely, the revolution frequency $f_{0}$ is determined for a fictitious particle with constant momentum error $\delta p / p_{0} c=\delta_{s}=$ DELTAP which travels along the design orbit. The RF frequencies are then set to $h f_{0}$.

If the machine contains at least one RF cavity, and if synchrotron radiation is on (Section 5.2), the EMIT command computes the equilibrium emittances and other electron beam parameters using the method of A. Chao [7]. In this calculation the effects of quadrupoles, sextupoles, and octupoles along the closed orbit is also considered. Thin multipoles are ignored, unless they have a fictitious length LRAD different from zero.

If the machine contains no RF cavity, or if synchrotron radiation is off, it only computes the parameters which are not related to radiation. Example:

```
RFC: RFCAVITY,HARMON...,VOLT=...
    BEAM, ENERGY=100.0, RADIATE
    EMIT,DELTAP=0.01
```


### 5.8.2 Representations of Beam with Full Coupling

The three commands

```
EIGEN,SAVE=name
ENVELOPE,SAVE=name,SIGMAO=name,LINE=line
TWISS3, SAVE=name, LINE=1ine
```

track different representations of the beam in three degrees of freedom. If the ring contains RF cavities, the command EMIT should be run before any of these commands in order to adjust the RF frequencies. Their actions of are the following:

EIGEN Tracks the eigenvectors for the three eigen-modes and prints the principal phase for each of them. Example:

```
RFC: RFCAVITY,HARMON...,VOLT=...
    BEAM,ENERGY=100.0,RADIATE
    EMIT,DELTAP=0.01
    EIGEN,SAVE=name
```

ENVELOPE Tracks the beam envelope in a way similar to the TRANSPORT program. This uses the eigenvectors and emittances for the three modes to determine the initial beam envelope, and tracks this envelope along the ring. Example:

```
RFC: RFCAVITY,HARMON...,VOLT=...
    BEAM,ENERGY=100.0,RADIATE
    EMIT,DELTAP=0.01
    ENVELOPE,SAVE=name,SIGMAO=name, LINE=line
```

TWISS3 Lists the coupled lattice functions defined in two papers by H. Mais [22] and G. Ripken [25]. These are essentially the projections of the lattice functions for the eigenmodes on the three planes. Example:

```
RFC: RFCAVITY,HARMON...,VOLT=...
    BEAM,ENERGY=100.0,RADIATE
    EMIT,DELTAP=0.01
    TWISS3,SAVE=name,LINE=line
```

The attributes mean:
SAVE If this keyword is present, it requests saving of the computed functions in a table. It may be followed by a name to be given to the table. If it appears without a value the table has the same name as the command. If the SAVE keyword is omitted, no table is generated. If the table is to be processed outside MAD, it must be written to disk using the ARCHIVE command (Section 7.5). The positions appearing in the tables are selected by means of the command

```
SELECT,OPTICS,RANGE=...
```

in a way similar to the OPTICS command.
SIGMAO May contain the name of a SIGMAO block, to be used as the initial beam envelope in ENVELOPE. in ENVELOPE or TWISS3.

For all three commands printing and selection for saving is controlled by the PRINT command (5.3). A SIGMAO block can be generated by two means. First, it may be entered like any definition:

```
name:SIGMAO, X,PX,Y,PY,T,PT,DX,DPX,DY,DPY,DT,DPT,&
    SIGX,R21,R31,R41,R51,R61,SIGPX,R32,&
    R42,R52,R62,SIGY,R43,R53,R63,SIGPY,&
    R54,R64,SIGT,R65,SIGPT
```

The first group of six values defines the initial orbit. For a static machine (constant energy) the second group of six values defines the initial dispersion. The values SIGxx are the standard deviations and the Rij the correlations defining the beam ellipsoid in TRANSPORT sense.

A SIGMAO block can also be generated and used as follows:

```
USE,RING
SAVESIGMA,LABEL=SIGMAHERE,PLACE=HERE
EMIT ! required only if RF cavities are present
ENVELOPE
USE,RING,RANGE=HERE/THERE
ENVELOPE,SIGMAO=SIGMAHERE
```

The first envelope command computes the periodic beam envelope for the line RING, and fills in the SIGMAO-block with the name SIGMAHERE. The second ENVELOPE uses the values in position HERE to track the envelope from HERE to THERE.

### 5.9 DYNAMIC and STATIC, Lie-Algebraic Analysis

The two commands

| DYNAMIC | DELTAP=real, MAP, ORBIT,A,N,\& RESONANCE, EXPONENT, HAMILTON, INVARIANT |
| :---: | :---: |
| STATIC, | DELTAP=real, MAP, ORBIT, FIXED, T, A, N, \& RESONANCE, EXPONENT, HAMILTON, BETATRON, \& NONLINEAR, CONJUGATE, INVARIANT |

both evaluate the fourth-order Lie transformation for one turn and print the eigenvectors and lattice parameters at the end of the system. Both have various options to print selected maps and functions. The differences are the following:

DYNAMIC The RF cavities are adjusted as for the command NORMAL. The transfer map is interpreted as dynamic, i.e. there is synchrotron motion with an average momentum error DELTAP.

STATIC The program assumes that there are no cavities, and it interprets the transfer map as static, i.e. having constant momentum error DELTAP.

Both commands have the following logical flags:
MAP Print the original map to be analysed.

ORBIT Print the map around the closed orbit.
Print the conjugating map (the transformation from the closed orbit to normal form).
N $\quad$ Print the normal form map.
RESONANCE Print the resonance coefficients left in the normal form.
EXPONENT Print the exponent for the normal form.
HAMILTON Print the Hamiltonian for the normal form.
INVARIANT Print the linear invariants.
The DYNAMIC command adjusts the RF frequencies according to the revolution frequencies. It uses the following attributes for this purpose:

RFCAVITY The name of a class of cavities whose phase lag is to be adjusted to make the time lag of the closed orbit approximately zero.

DELTAP The average momentum error $\Delta p / p_{0} c$ for the closed orbit.
The STATIC command also finds the fixed point of the map, that is the variation of the closed orbit with momentum. It has the following additional attributes:

DELTAP The constant momentum error $\Delta p / p_{0} c$ for the closed orbit.
FIXED Print the map about the fixed point.
T Print the transformation to the fixed point.
BETATRON Print the betatron map.
NONLINEAR Print the non-linear factor of the betatron map.
CONJUGATE Print the map conjugate to the betatron factor.

### 5.10 OPTICS Command

Control system applications often require tables with arbitrary selections of element data and/or lattice functions. Such tables can be generated by the command

```
OPTICS, BETX=real,ALFX=real,MUX=real,&
    BETY=real,ALFY=real,MUY=real,&
    DX=real,DPX=real,DY=real,DPY=real,&
    X=real,PX=real, Y=real, PY=real,T=real,PT=real&
    WX=real,PHIX=real,DMUX=real,&
    WY=real,PHIY=real,DMUY=real,&
    DDX=real,DDY=real,DDPX=real,DDPY=real,&
    LINE=beam-line, BETAO=name, CENTRE,&
    FILENAME=file-name,DELTAP=real:real:real,&
    COLUMNS=name{, name}
```

It first creates a table in memory. It then writes it on a disk file in coded TFS format (see Appendix C). Initial conditions for the optical functions are specified like for the TWISS command. They are explained in Sections 1.5 and 5.6. Further attributes are:

CENTRE Normally output occurs at the exit of each selected element. If the CENTRE flag is on, output occurs at the centre of each selected element;

FILENAME The output is written on the file file-name (default: optics). Note that the resulting table is written on this file and erased from the computer's memory. In order to plot it, it must be read back by an RETRIEVE command.

COLUMNS Up to 50 table columns may be selected by name for output. All optical and chromatic functions listed in Sections 1.5.1, 1.5.3, and 1.5.4 are accepted. Further possibilities are:

NAME The element name.
CLASS The smallest class containing the element as an object.
KEYWORD The element keyword.
TYPE The TYPE attribute of the element.
S
DP The current $\Delta E / p_{s}$. It is not called DELTAP to avoid conflicts with the command attribute of this name which selects a single value for the calculation.

L The element length.
RADLOSS The systematic part of the relative radiation loss for this element.
KOL The dipole component of this element (excluding kicks).
$\mathrm{KnL} \quad$ The integrated multipole component. The value of $n$ specifies the multipole with $2 n+2$ poles, and $0 \leq n \leq 9$.
VOLT The cavity voltage.
LAG The RF frequency lag.
FREQ The RF frequency.
HARMON The RF harmonic number.
TILT The roll angle of the element, zero for multipoles.
KS The integrated solenoid strength.
HKICK The horizontal corrector deflection.
VKICK The vertical corrector deflection.
E1 The entrance pole face angle for dipoles.
E2 The exit pole face angle for dipoles.
H1 The entrance pole face curvature for dipoles.
H2 The exit pole face curvature for dipoles.
EFIELD The electrostatic field for a separator.
Example:

```
! Define element classes for a simple cell:
B: SBEND,L=35.09, ANGLE = 0.011306116
QF: QUADRUPOLE,L=1.6,K1=-0.02268553
QD: QUADRUPOLE,L=1.6,K1=0.022683642
SF: SEXTUPOLE,L=0.4,K2=-0.13129
SD: SEXTUPOLE,L=0.76,K2=0.26328
```

! Define the cell as a sequence:
CELL: SEQUENCE

```
B1: B, AT=19.115
SF1: SF, AT=37.42
QF1: QF, AT=38.70
B2: B, AT=58.255,ANGLE=B1[ANGLE]
SD1: SD, AT=76.74
QD1: QD, AT=78.20
ENDM: MARKER, AT=79.0
ENDSEQUENCE USE,CELL
SELECT, OPTICS, SBEND, QUAD,SEXT
OPTICS,FILENAME="cell.optics.f",EXIT,COLUMN=NAME,S,BETX,BETY
```

The resulting table file is listed in Table 5.3.

Table 5.3: An OPTICS Output Table Example

| © GAMTR | \%f 64.3336 |  |  |
| :---: | :---: | :---: | :---: |
| © ALFA | $0.241615 \mathrm{E}-03$ |  |  |
| © XIY | -. 455678 |  |  |
| © XIX | 2.05279 |  |  |
| © QY | 0.250049 |  |  |
| © QX | 0.249961 |  |  |
| © CIRCUM | 79.0000 |  |  |
| © DELTA | $0.000000 \mathrm{E}+00$ |  |  |
| © COMMENT | \%20s "DATA FOR TEST CELL" |  |  |
| © ORIGIN | $\% 24 s$ "MAD 8.01 IBM - VM/CMS" |  |  |
| © DATE | \%08s "19/06/89" |  |  |
| © TIME | \%08s "09.47.40" |  |  |
| * NAME | S | BETX | BETY |
| , ) $\% 16 \mathrm{~s}$ | \%f | \%f | \%f |
| B1 | 36.6600 | 24.8427 | 126.380 |
| SF1 | 37.6200 | 23.8830 | 130.925 |
| QF1 | 39.5000 | 23.6209 | 132.268 |
| B2 | 75.8000 | 124.709 | 25.2153 |
| SD1 | 77.1200 | 130.933 | 23.8718 |
| QD1 | 79.0000 | 132.277 | 23.6098 |

### 5.11 Examples for SPLIT and OPTICS Commands

The following is an excerpt of the LEP description:

```
! Bending magnet pairs:
! The definitions take into account the different magnetic length
! for the inner and outer pairs of a group of six.
B2: RBEND, L=11.55,ANGLE=KMB2,K1=KQB,K2=KSB, &
    E1=-.25*B2[ANGLE],E2=- . 25*B2[ANGLE]
B20UT : B2, ANGLE=1.00055745184472*KMB2, &
    E1=- .25*B20UT[ANGLE], E2=- .25*B20UT [ANGLE]
B2MID: B2, ANGLE=1.00111490368947*KMB2, &
    E1=- .25*B2MID[ANGLE],E2=- .25*B2MID [ANGLE]
! Quadrupoles:
MQ: QUADRUPOLE,L=1.6 ! standard quadrupoles =
QD: MQ, K1=KQD ! cell quadrupoles, defocussing
QF: MQ, K1=KQF !cell quadrupoles, focussing
! Sextupoles:
MSF: SEXTUPOLE, L=0.40 ! F sextupoles
MSD: SEXTUPOLE, L=0.76 ! D sextupoles
SF1.2: MSF, K2=KSF1.2 ! F family 1, circuit 2
SF2.2: MSF, K2=KSF2.2 ! F family 2, circuit 2
SF3.2: MSF, K2=KSF3.2 ! F family 3, circuit 2
SD1.2: MSD, K2=KSD1.2 ! D family 1, circuit 2
SD2.2: MSD, K2=KSD2.2 ! D family 2, circuit 2
SD3.2: MSD, K2=KSD3.2 ! D family 3, circuit 2
! Orbit correctors and monitors:
CH: HKICK, L=0.4 ! Horizontal orbit correctors
CV: VKICK, L=0.4 ! Vertical orbit correctors
MHV: MONITOR, L=0 ! Orbit position monitors
LEP:SEQUENCE
```

```
QF23.R1: QF, AT=639.180037
```

QF23.R1: QF, AT=639.180037
SF2.QF23.R1: SF2.2, AT=640.460037
SF2.QF23.R1: SF2.2, AT=640.460037
B2L.QF23.R1: B20UT, AT=647.257037
B2L.QF23.R1: B20UT, AT=647.257037
B2M.QD24.R1: B2MID, AT=659.147037
B2M.QD24.R1: B2MID, AT=659.147037
B2R.QD24.R1: B20UT, AT=671.037037
B2R.QD24.R1: B20UT, AT=671.037037
CV.QD24.R1: CV, AT=677.392037, KICK=KCV24.R1
CV.QD24.R1: CV, AT=677.392037, KICK=KCV24.R1
PU.QD24.R1: MHV, AT=677.712037
PU.QD24.R1: MHV, AT=677.712037
QD24.R1: QD, AT=678.680037
QD24.R1: QD, AT=678.680037
SD2.QD24.R1: SD2.2, AT=680.140037
SD2.QD24.R1: SD2.2, AT=680.140037
B2L.QD24.R1: B20UT, AT=686.757037
B2L.QD24.R1: B20UT, AT=686.757037
B2M.QF25.R1: B2MID, AT=698.647037
B2M.QF25.R1: B2MID, AT=698.647037
B2R.QF25.R1: B20UT, AT=710.537037
B2R.QF25.R1: B20UT, AT=710.537037
CH.QF25.R1: CH, AT=716.942037, KICK=KCH25.R1
CH.QF25.R1: CH, AT=716.942037, KICK=KCH25.R1
QF25.R1: QF, AT=718.180037
QF25.R1: QF, AT=718.180037
SF1.QF25.R1: SF1.2, AT=719.460037
SF1.QF25.R1: SF1.2, AT=719.460037
B2L.QF25.R1: B20UT, AT=726.257037
B2L.QF25.R1: B20UT, AT=726.257037
B2M.QD26.R1: B2MID, AT=738.147037
B2M.QD26.R1: B2MID, AT=738.147037
B2R.QD26.R1: B20UT, AT=750.037037

```
B2R.QD26.R1: B20UT, AT=750.037037
```

```
CV.QD26.R1: CV, AT=756.392037, KICK=KCV26.R1
PU.QD26.R1: MHV, AT=756.712037
QD26.R1: QD, AT=757.680037
SD1.QD26.R1: SD1.2, AT=759.140037
B2L.QD26.R1: B20UT, AT=765.757037
B2M.QF27.R1: B2MID, AT=777.647037
B2R.QF27.R1: B20UT, AT=789.537037
QF27.R1: QF, AT=797.180037
SF3.QF27.R1: SF3.2, AT=798.460037
B2L.QF27.R1: B20UT, AT=805.257037
B2M.QD28.R1: B2MID, AT=817.147037
B2R.QD28.R1: B20UT, AT=829.037037
PU.QD28.R1: MHV, AT=835.712037
QD28.R1: QD, AT=836.680037
SD3.QD28.R1: SD3.2, AT=838.140037
B2L.QD28.R1: B20UT, AT=844.757037
B2M.QF29.R1: B2MID, AT=856.647037
B2R.QF29.R1: B20UT, AT=868.537037
CH.QF29.R1: CH, AT=874.942037, KICK=KCH29.R1
ENDSEQUENCE
```

In the above structure it is easy to select many sets of observation points:

- Print at all F sextupoles:

```
PRINT,MSF
```

- Split all quadrupoles at $1 / 3$ of their length for OPTICS command:

```
SPLIT, QUADRUPOLE, FACTOR=1/3
```

- Misalign two quadrupole QF25.R1 and QD26.R1:

```
EALIGN,QF25.R1,QD26.R1,DX=0.001*GAUSS(),DY=0.0005*GAUSS()
```

- Print first-order matrices for elements B2L.QD24.R1 through CV.QD26.R1:

```
SELECT,FIRST,B2L.QD24.R1[1]/CV.QD24.R1[1]
```

Print lattice functions at all F-sextupoles of the first family, if connected to the second circuit:
PRINT,SF1. 2

# Chapter 6. Calculation of Beam Parameters 

Table 6.1: Electron Beam Parameter Command

| Name | Function | Section |
| :--- | :--- | :--- |
| BMPM | Calculate and print beam parameters | 6.2 |

### 6.1 Introduction

The program BEAMPARAM (see [17]) which served as the basis for this module, is valid for $\mathrm{e}^{+}-\mathrm{e}^{-}$storage rings only. Inside MAD, however, this new module will use the correct mass and radius of the proton if this is chosen by the user. This means that all formulae will remain correct even in the case of the proton. However, it has not been checked whether some of them have been derived under the assumption that the particles are electrons. In the text, only $\mathrm{e}^{+}-\mathrm{e}^{-}$storage rings are mentioned.

The utility of the BMPM command is achieved at the price of a few simplifying assumptions and approximations concerning the accelerator lattice. Although these conditions are quite well satisfied by many $\mathrm{e}^{+}-\mathrm{e}^{-}$storage rings, the user does need to be aware of them. Accordingly we list the most important among them below:

Flat machine If any element has a non-zero TILT parameter then the command will abort. This excludes, in particular, skew quadrupoles and vertical bends, hence the vertical dispersion will be zero everywhere. Combined-function dipoles are not accepted.

Emittance coupling Although the previous assumption implies that the betatron coupling vanishes so that the vertical emittance will be determined by the small transverse components of the photon recoil, the vertical emittance is in fact calculated using a non-zero betatron coupling parameter $\kappa$ (KAPPA). This factor is defined by the equation $E_{y} / E_{x}=\kappa^{2}\left(J_{x} / J_{y}\right)$.

Damping partition number variation In reality, this is achieved by small shifts of the equilibrium momentum which place the beam on a slightly different orbit. BMPM computes the values of the damping partition numbers on the central orbit with Eqn. 6.1-Eqn. 6.1. These are later used in the calculations of damping times, emittances and energy spread. In reality, their values on other (off-momentum) orbits involve further synchrotron radiation integrals. However, changes in the damping partition number are effected by artificially changing the value of the synchrotron integral $I_{4}$. For convenience, it is the value of the ratio $I_{4} / I_{2}$ that is specified in the input rather than $I_{4}$ itself. It is the user's responsibility to ensure that the damping partition number variation brought about in this way does not violate the aperture constraints of his machine.

$$
\begin{array}{ll}
\text { Horizontal } J_{x} & =1-I_{4} / I_{2} \\
\text { Vertical } J_{y} & =1 \\
\text { Energy } J_{e} & =2+I_{4} / I_{2}
\end{array}
$$

Luminosity The luminosity value computed assumes that the beam sizes have their natural values, and does not allow for any beam-beam blow-up.

### 6.2 BMPM Command

The BMPM command performs the calculation of beam parameters for an $\mathrm{e}^{+}-\mathrm{e}^{-}$storage ring according to the parameters and options specified below. It requires the prior execution of the commands BEAM and TWISS with SAVE option. This command requires the beam energy to be set, and the machine
to contain an RF cavity whose shunt impedance and filling time are specified. It does not handle combined function dipoles.

```
BMPM, NINT=integer,DELQ=real,TAUQ=real,BUCKET=real,&
    KAPPA=real,I4I2=real,EXDATA=real,FX=real,FY=real,&
    KHM=real,SYNRAD=logical,CLORB=logical,&
    TOUSCH=logical,SINGLE=logical,EVARY=logical,&
    MIDARC=logical, INTERACT=range, RANGE=range
```

NINT Number of crossing points (default: twice the number of bunches).
DELQ Maximum permissible beam-beam tune shift per crossing (default: 0.06 ).
TAUQ Quantum lifetime (default: 600 Minutes).
BUCKET $\quad \sigma_{b} / \sigma_{e}$, where $\sigma_{b}$ is the bucket half-height, and $\sigma_{e}$ the relative r.m.s. energy spread (default: 0 ).

KAPPA Coupling factor between horizontal and vertical betatron oscillations (default: 0 ).
I4I2 Ratio of the two synchrotron integrals $I_{4}$ and $I_{2}$ (default: 0 ).
EXDATA Horizontal emittance $E_{x}$ (default: 0 ) in $\mu \mathrm{m}$.
FX Horizontal multiplication factor (default: 10).
FY Vertical multiplication factor (default: 10).
KHM Higher mode loss factor (default: 0 ) in $\mathrm{V} / \mathrm{pC}$.
SYNRAD Performs synchrotron radiation calculations (default: F).
CLORB Performs closed orbit calculations (default: F).
TOUSCH Performs Touschek lifetime calculations (default: F).
SINGLE True for one beam, false for two beams (default: F).
EVARY Vary energy between current nominal energy and twice its value until the calculated power is equal to the power given (default: F).

MIDARC If MIDARC=T, then the $\mathrm{e}^{-}$and $\mathrm{e}^{+}$are assumed to be interleaved with equal spacing; otherwise they are assumed to be coincident. The small difference in arrival times which occurs in practice is neglected. For single beams, the MIDARC parameter is irrelevant (default: F).

INTERACT Position (name, number, etc.) of the interaction element. If a range is entered, the first element of the range is used (default: \#E, see Section 4.7).

RANGE Element range for which a detailed table is printed (default: no range, see Section 4.7).
Example:
xxx: RFCAVITY,SHUNT=...,TFILL=...
BEAM, ENERGY=100.0
TWISS, SAVE BMPM

### 6.3 Output of overall machine parameters

### 6.3.1 Synchrotron integrals

This table gives the synchrotron integrals $I_{1}$ to $I_{5}$ as defined in reference [18]. Further integrals, $I_{6 x}$ and $I_{6 y}$, defined by M.Bassetti, as well as $I_{8}$ are also calculated and printed.

### 6.3.2 Machine parameters

This table contains the tunes $Q_{x}, Q_{y}$, the betatron functions $\beta_{x}^{*}$ and $\beta_{y}^{*}$ and the dispersions $D_{x}^{*}$ and $D_{y}^{*}$ at the crossing point. Since the program does not handle vertical dispersion, $D_{y}^{*}$ is set to zero. Also printed are the momentum compaction $\alpha$, the machine circumference $C$, the revolution time $t_{0}$, the damping partition numbers $J_{x}, J_{y}, J_{e}$, the derivative of $J_{x}$ with respect to $d E / E$, then $d E / E$ itself, and finally the frequency change $d f_{R F}$.

### 6.3.3 Beam parameters and luminosities

This table gives the beam energy, the coupling, the larger of the horizontal and vertical tune shifts $\Delta Q$, the energy loss per turn due to synchrotron radiation $U_{0}$, the r.m.s. energy spread $\sigma_{e}$, the magnetic rigidity $B \rho$, and the damping times for the horizontal and vertical betatron oscillations, $\tau_{x}$ and $\tau_{y}$, and for the synchrotron oscillations $\tau_{e}$.
$\sigma_{x}^{*}$ and $\sigma_{y}^{*}$ are the r.m.s. beam radii at the crossing point, defined by INTERACT:

- $\sigma_{x 0}^{*}$ and $\sigma_{y 0}^{*}$ are the uncoupled betatron values.
- $\sigma_{x c}^{*}$ and $\sigma_{y c}^{*}$ are the values due to coupled betatron oscillations.
- $\sigma_{x T}^{*}$ and $\sigma_{y T}^{*}$ are the total values, including coupled betatron oscillations and energy oscillations.

Convention for the coupling $\kappa$ : If a non-zero value of the coupling parameter $\kappa$ is read in, it is used for calculating $\sigma_{x c}^{*}, \sigma_{y c}^{*}, \sigma_{x T}^{*}, \sigma_{y T}^{*}, E_{x c}, E_{y c}$ etc. and never gets changed. In the output $\kappa$ is labelled (D) KAPPA.
If the input value $\kappa=0$, BMPM computes $\kappa$ so that the beam-beam tune shifts $\Delta Q_{x}$ and $\Delta Q_{y}$ vary with the current as shown schematically in Figure 6.1. For currents at the beam-beam limit $\Delta Q$ and above, the coupling $\kappa$ is adjusted so that $\Delta Q_{x}=\Delta Q_{y}$ by having $\sigma_{x T}^{*} / \sigma_{y T}^{*}=\beta_{x}^{*} / \beta_{y}^{*}$. (This is usually called optimum coupling.) For currents below the beam-beam limit $\Delta Q$, the coupling $\kappa$ is adjusted so that $\Delta Q_{x}<\Delta Q$ but $\Delta Q_{y}=\Delta Q$.

Beam current Three different methods are foreseen for calculating the circulating current $I$, the stored number of particles $N$ in each beam, and the luminosity $L$.

1. If neither a value for the current $I_{x y}$ nor a value for the RF generator power $P_{g}$ is given in the data, i.e. $I_{x y}=P_{g}=0$, then the currents $I_{x}$ and $I_{y}$ are calculated so that $\Delta Q_{x}=\Delta Q$ and $\Delta Q_{y}=\Delta Q$ respectively. These currents are then used to calculate the stored number of particles $N_{x}$ and $N_{y}$ and the luminosities $L_{x}$ and $L_{y}$. For optimum coupling, $I_{x}=I_{y}, N_{x}=N_{y}$ and $L_{x}=L_{y}$. This option is typical for storage rings in the energy range where the luminosity is limited by the beam-beam tune shift.
2. If a value for the current $I_{x y}$ is specified in the data, i.e. $I_{x y} \neq 0$, then $I_{x}=I_{y}=I_{x y}$ and these values are used to calculate the beam-beam tune shifts $\Delta Q_{x}$ and $\Delta Q_{y}, N_{x}$ and $N_{y}\left(=N_{x}\right)$ and the luminosities $L_{x}$ and $L_{y}\left(=L_{x}\right)$. This option typically applies to storage rings in the energy range where the current is limited, and to synchrotrons. The $\Delta Q$ printed is the larger of $\Delta Q_{x}$ and $\Delta Q_{y}$, it may well exceed the beam-beam limit.


Figure 6.1: Variation of the beam-beam tune shifts $\Delta Q_{x}$ and $\Delta Q_{y}$ with the current obtained by varying the coupling $\kappa$
3. If an RF generator power $P_{g} \neq 0$ is specified in the data, the current $I_{x}$ is adjusted so that the RF generator power necessary to sustain it is equal to the input value $P_{g}$. The current $I_{x}=I_{y}$ is then used to calculate $\Delta Q_{x}$ and $\Delta Q_{y}, N_{x}$ and $N_{y}\left(=N_{x}\right), L_{x}$ and $L_{y}\left(=L_{x}\right)$.

In the output, the input parameters, $\Delta Q, I_{x}, P_{g}$, are labelled (D) and the calculated parameters (C).

The beam-beam bremsstrahlung lifetime $\tau_{b b}$ is calculated according to [30], using the bucket height obtained in the calculation of the RF parameters, and assuming that there are $N_{\text {int }}$ identical crossings. For machines with different crossings the calculated $\tau_{b b}$ must be scaled appropriately.

### 6.3.4 RF related parameters

The RF parameter calculation is done in two different ways, depending on the value of the RF generator power in the data. If $P_{g}=0$ there are two possible values for the current $I_{x}$ : either $P_{g}=I_{x y}=0$, in which case a value for $I_{x}$ is calculated, or $P_{g}=0$ and $I_{x y} \neq 0$ in which case $I_{x}=I_{x y}$. Once one has a value for $I_{x}$ then a single-pass calculation is sufficient to obtain all the output parameters described below. If $P_{g} \neq 0$, the current $I_{x}$ is adjusted so that the input value and the calculated value of $P_{g}$ agree, before the rest of the output parameters is calculated.

The output contains the harmonic number $f_{R F} / f_{0}$, the peak RF voltage $V_{R F}$, the stable phase angle $\phi_{s}$, the synchrotron tune $Q_{s}$ and the synchrotron frequency $f_{s}$, the half-height of the bucket $\sigma_{b}$ in units of $\sigma_{e}$, the r.m.s. bunch length $\sigma_{s}$, the quantum lifetime $\tau_{q}$ and the Touschek lifetime $\tau_{1 / 2}$ calculated according to [27, 8, 28].

If $\gamma \sigma_{x^{\prime}} / \sigma_{b}>100$ then we use an approximation which is good to within $2 \%$ [28], otherwise we use the exact formula. Since the approximation involves an integration along the orbit, it is somewhat time-consuming. The exact formula will take even longer as a double integral is involved. So, it is preferable not to ask for the Touschek lifetime unless it is really needed.

Convention for RF frequency $f_{R F}$ : The harmonic number $f_{R F} / f_{0}$ is computed such that it is the multiple of $k_{b}$ which gives an RF frequency $f_{R F}$ closest to the input value.

Convention for RF voltage $V_{R F}$ : The RF voltage $V_{R F}$ may be obtained in one of three ways. Firstly it may be read in as data as an attribute of the RF cavities. Secondly it may be
computed such that an energy loss $\left(U_{0}+k_{h m} Q_{b}\right)$ is compensated, at the specified quantum lifetime $\tau_{q}$, where $Q_{b}$ is the bunch charge and $k_{h m}$ is the higher mode loss factor. Finally it can be calculated from the condition that the bucket half height $\sigma_{b}$ is BUCKET times the r.m.s. energy spread $\sigma_{e}$
The computation of the RF generator power $P_{g}$ involves a complete beam-loading calculation according to [29]. The input data required are: the length of the active RF system $L_{c}$ (sum of physical lengths of all cavities), its shunt impedance per unit length $Z$, and its unloaded filling time $T_{\text {fill }}$ (all attributes of the RF cavities). The beam is described by parameters which are already known-such as the current $I_{x}$ and the number of bunches $k_{b}$-and by the logical variables SINGLE and MIDARC. If SINGLE=T, then the calculation is done for a single beam circulating in the machine; otherwise the calculation is done for two counter-rotating beams of $\mathrm{e}^{-}$and $\mathrm{e}^{+}$. The arrival times of the $\mathrm{e}^{-}$and $\mathrm{e}^{+}$ bunches at the RF cavities are controlled by the variable MIDARC. If MIDARC=T, then the $\mathrm{e}^{-}$and $\mathrm{e}^{+}$are assumed to be interleaved with equal spacing; otherwise they are assumed to be coincident. The small differences in arrival times which occurs in practice [4] is neglected. For single beams, the MIDARC parameter is irrelevant.

The two cavity attributes BETRF and LAG, represent the RF coupling parameter $\beta_{R F}$ and the cavity tuning angle $\psi_{R F}$ respectively. If both $\beta_{R F}$ and $\psi_{R F}$ are zero, BMPM will adjust them such that the RF generator power, $P_{g}$, is minimised. If $\beta_{R F}$ is different from zero, $\psi_{R F}$ will be varied such that the RF generator power $P_{g}$ is minimised. If both are different from zero, no such optimisation takes place.

The output of the beam loading calculation consists of $\beta_{R F}, \psi_{R F}, \tau_{q}$ and BUCKET, all preceded by (C) or (D) accordingly.

### 6.4 Evaluation of the beam size

If a valid element range is given (RANGE), a table of beam sizes is printed for the elements in the range. For each element, it gives the number, name, $\eta, \eta^{\prime}, \beta_{x}, \beta_{y}, \alpha_{x}, \alpha_{y}, F_{x} \sigma_{x}$, and $F_{y} \sigma_{y}$, all taken at the entrance of the element. The values of $F_{x}$ and $F_{y}$ are input data (FX and FY).

Convention for beam radii: The horizontal beam radius $\sigma_{x}$ is calculated for an uncoupled beam using the emittance $E_{x 0}$, the vertical one for a fully coupled beam using the emittance $E_{x 0} / 2$.

Because of this convention, the actual beam sizes for all amounts of coupling should be smaller than or equal to the figures printed. It may be found useful to split elements with strong variations of $\beta_{x}$ and $\beta_{y}$, e.g. low- $\beta$ quadrupoles, into several pieces in the data in order to get better results for the beam size variation in these elements.

### 6.5 Synchrotron radiation, Calculated for One Beam

If in addition to a valid range, $\operatorname{SYNRAD=T,~the~table~of~beam~sizes~will~contain~in~addition~for~each~}$ element the horizontal beam divergence $\sigma_{x^{\prime}}$, the vertical beam divergence $\sigma_{y^{\prime}}$, the bending radius $\rho$, the critical photon energy $E_{c}$, the synchrotron radiation power/metre, and the number of photons $/ \mathrm{m} / \mathrm{s} / \mathrm{keV}$ at the critical energy.
Convention for divergences: $\sigma_{x^{\prime}}$ is calculated for an uncoupled beam; $\sigma_{y^{\prime}}$ is calculated for a fully coupled beam; $\rho, E_{c}$ etc. are only calculated for bending magnets ( $\phi \neq 0$ ) or quadrupoles ( $K_{1} \neq 0$ ). For all these calculations, combined function magnets are not permitted ( $K_{1} \phi \neq 0$ ). In quadrupoles, $\rho, E_{c}$ etc., as defined in [20], are calculated for a fully coupled beam at a distance $\sigma$ from the quadrupole axis.
Again, it may be found useful to split elements with strong variations of $\beta_{x}$ and $\beta_{y}$, e.g. low- $\beta$ quadrupoles, into several pieces in the data in order to obtain better results for the variation of the synchrotron radiation parameters in those elements.

### 6.6 Closed orbit calculations

The sensitivity of the closed orbit to alignment errors of the quadrupoles, and excitation errors and tilts of the bending magnets can be expressed by integrals around the machine circumference of appropriate powers of the amplitude functions $\beta_{x}$ and $\beta_{y}$, and the focussing parameter $K_{1}$.

If in addition to a valid range, CLORB $=\mathrm{T}$, the table of beam sizes will contain in addition for each element the amplification factors $P_{x}$ and $P_{y}$ for all quadrupoles and bending magnets. For quadrupoles, $P_{x}$ and $P_{y}$ are amplification factors for horizontal and vertical misalignments. They are the ratio between the closed orbit distortion in that element, which will not be exceeded in $98 \%$ of all machines, to the r.m.s. displacement of the quadrupoles, assumed to be the same for all quadrupoles. For bending magnets, $P_{x}$ is the $98 \%$ ratio between closed orbit distortions in metres and relative r.m.s. magnet errors $\Delta B / B$, and similarly $P_{y}$ is the ratio between closed orbit distortions in metres and the r.m.s. tilt of the magnets.

## Chapter 7. File and Pool Handling

Table 7.1: File Handling Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| ASSIGN | Assign standard streams to files | 7.2 |
| SAVE | Save machine structure in MAD format | 7.3 |
| CALL | Read alternate input file | 7.3 |
| RETURN | Return to calling file | 7.3 |
| EXCITE | Read element excitations | 7.4 |
| INCREMENT | Increment element excitations | 7.4 |
| ARCHIVE | Archive an internal table | 7.5 |
| RETRIEVE | Retrieve an internal table | 7.5 |
| PACKMEMORY | Garbage removal | 7.6 |
| STATUS | Show status of files | 7.7 |
| POOLDUMP | Dump data pool to disk | 7.8 |
| POOLLOAD | Reload data pool from disk | 7.8 |

### 7.1 I/O Data Sets (Files)

In MAD most data streams are referred to by their name and opened by FORTRAN OPEN statements with a file name. Only the standard input and output are referred to by number and must be assigned to devices by external commands (job control language, execute files or the like). The standard data streams used in MAD are listed in Table 7.2. The format of file names is described in Appendix B for various computer operating systems.

Table 7.2: Standard Files Used by MAD

| Purpose | unit | file name |
| :--- | :--- | :--- |
| Command dictionary input | 4 | dict |
| Normal input | 5 |  |
| Input lines and error messages | 6 |  |
| Plot output (GKS metafile) | 8 | mad.metafile |
| Plot output (HIGZ metafile) | 8 | mad.ps |
| Normal output | 14 | print |
| Dynamic tables | 15 | table |

### 7.2 ASSIGN Statement

The ASSIGN statement is able to reroute three of the standard I/O streams to different files:
ASSIGN, DATA=file-name, ECHO=file-name,PRINT=file-name

For each stream=file-name clause, it connects the stream stream to the file file-name. The special file name terminal denotes the interactive terminal. Example:

ASSIGN, ECHO=PRINT, PRINT=TERMINAL
This example sends the ECHO stream to the standard PRINT file, and the PRINT stream to the terminal. Standard connections can be reestablished by

### 7.3 SAVE, CALL, and RETURN Statements

The SAVE command

```
SAVE, FILENAME=string, PATTERN=string
```

causes all beam element, beam line, and parameter definitions to be written on the named file. The output format is similar to the normal input format. The file may be read again in the same run. The attributes are
filename The name of the file to be written (default: save).
PATTERN A "wild-card" pattern permitting selective saving
The default is to save all definitions; this is equivalent to the PATTERN=".*". For definition of the pattern refer to Section 1.7. Examples:

```
SAVE,FILENAME="structure"
SAVE,FILENAME="quadrupoles",PATTERN="Q.."
SAVE,FILENAME="strengths",PATTERN="K.*QD.*R1"
```

The first command saves all definitions onto file structure. The second command saves all definitions whose names begin with the letter $Q$ and have exactly three characters onto file quadrupoles. The third command saves all definitions whose names begin with $K$, contain QD, and end with .R1. The two occurrences of .* replace arbitrary strings of any length (including zero); and $\backslash$. replaces a literal period character.

The command The caLL command

CALL, FILENAME=string
serves to read an alternate input file. Input continues on that file until a RETURN statement or end of file is encountered. The attribute is

FILENAME The name of the file to be read (default: save).
Example:

CALL, FILENAME=STRUCT
reads back the file written by the above SAVE example.
The command RETURN

```
RETURN
```

on an alternate input file causes reading to resume with the next line following the CALL statement on the standard input file. Both CALL and RETURN statements cannot be followed by any other information (except comments) on the same line. The CALL/RETURN structure can be nested up to a level 10 ; i.e. an alternate input file can issue another CALL. Circular calls are forbidden. The following example shows a possible sequence of input:

```
contents of contents of contents of
main input file file "STRUCT" file "ACTION"
    TITLE, "page header"
any commands
CALL, FILENAME=STRUCT
------------------------->
    beam element definitions
    beam line definitions
    parameter definitions
    RETURN
<--------------------------
more definitions
CALL, FILENAME=ACTION
-------------------------------------------------------
                                    action commands
                                    RETURN
<-------------------------------------------------------
more action commands
SAVE, FILENAME=STRUCT
STOP
```


### 7.4 Element Excitations

Two commands, EXCITE and INCREMENT allow to enter complete lists of element excitations:

```
EXCITE, FILENAME=file-name,NAME=name,VALUE=name
INCREMENT, FILENAME=file-name,NAME=name,VALUE=name
```

Both have the same attributes:
FILENAME The name of a TFS file (see Appendix C) to be read.
NAME The name of the table column containing the strength names.
VALUE The name of the table column containing the strength values (for EXCITE) or increments (for INCREMENT).

The names in the NAME column must refer to global MAD parameters. If a name is not known, MAD creates a new parameter of this name. The TFS table may contain other columns, but they will be ignored.

### 7.5 Archiving and Retrieving Dynamic Tables

### 7.5.1 ARCHIVE Command

ARCHIVE writes a dynamic table on a formatted TFS file (see Appendix C):

```
ARCHIVE,TABLE=name,FILENAME=file-name
```

The command has two attributes:
TABLE The name of a previously created table to be written. It may reside in memory or (partly) in bulk storage.

Example:
ARCHIVE,TABLE=TWISS,FILENAME=LEP.TWISS.F

### 7.5.2 RETRIEVE Command

RETRIEVE reads a formatted TFS file (see Appendix C) into memory and creates a dynamic table.

```
RETRIEVE,TABLE=name,FILENAME=file-name
```

The command has two attributes:
TABLE The name of a dynamic table to be created. It need not be the same as when the table was written. If there is already a table with this name, it is deleted before reading a new copy.

FILENAME The name of the file to be read (default: twiss).
Example:
RETRIEVE,TABLE=TWISS1,FILENAME=LEP.TWISS.F
reads file written by the ARCHIVE example and creates a new table TWISS1.

### 7.6 PACKMEMORY Command, Garbage Removal

The MAD memory management routines, based on the ZEBRA system, reclaim unused memory space automatically. One may however force garbage collection at any time by the PACKMEMORY command

```
PACKMEMORY
```


### 7.7 STATUS

The STATUS command
STATUS
causes MAD to list all known files on the message stream. These include all files which were referred to in the current program run.

### 7.8 Dumping and Reloading the Memory Pool

The POOLDUMP command
POOLDUMP, FILENAME=file-name
dumps the complete memory pool on the named disk file. Its attribute is:
FILENAME The name of the file to receive the dump (default: pooldump).
Memory dumped by POOLDUMP can be reloaded in a subsequent MAD run by a POOLLOAD command:

```
POOLLOAD,FILENAME=file-name
```

This may save considerable time compared to regenerating a complex data structure from input. Its attribute is:
FILENAME The name of the file to reload (default: pooldump).

## Chapter 8. Error Definitions

Table 8.1: Error Definition Commands

| Name | Function | Section |
| :--- | :--- | :--- |
| EALIGN | Specify misalignment(s) | 8.1 |
| EFCOMP | Specify field error(s) | 8.2 |
| EFIELD | Specify field error(s) | 8.2 |
| EOPT | Specify error options | 8.3 |
| EPRINT | List errors assigned to elements | 8.4 |

This chapter describes the commands which provide error assignment and output of errors assigned to elements. ${ }^{1}$ It is possible to assign alignment errors and field errors to single beam elements or to ranges of beam elements. Errors can be specified both with a constant or random values. Error definitions consist of four types of statements listed in Table 8.1. They may be entered after having selected a beam line by means of a USE command (see Section 5.1).

### 8.1 Misalignment Definitions

Alignment errors are defined by the EALIGN command. The misalignments refer to the local MAD reference system for a perfectly aligned machine (see Section 1.1). Possible misalignments are displacements along the three coordinate axes, and rotation about the coordinate axes. Alignment errors can be assigned to all beam elements except drift spaces. The effect of misalignments is treated in a linear approximation. Position monitors can be given read errors in both horizontal and vertical planes. Monitor read errors (MREX and MREY) are ignored for all other elements. Each new EALIGN statement replaces the misalignment errors for all elements in its range.

Alignment error values are defined by the statement

```
EALIGN, RANGE=range,TYPE=name,DX=real,DY=real,DS=real,&
    DPHI=real,DTHETA=real,DPSI=real,&
    MREX=real,MREY=real
```

with the attributes
RANGE A description of the beam elements in which the error occurs. It may contain up to five different ranges (see Section 4.7):
EALIGN, RANGE=QF , QD [2]/QD [4] , \#3/\#5,\#13/\#15

TYPE
Name(s) attached to physical element(s) by a TYPE=name clause (see Section 3). This may contain up to five different types
EALIGN , TYPE=MQ , MA , MB , MC , MD

RANGE and TYPE can be given in the same command. All elements will be affected which have TYPE=name or which are members of range.

DX The misalignment in the $x$-direction for the entry of the beam element (default: 0 m ). DX $>0$ displaces the element in the positive $x$-direction (see Figure 8.1).

[^1]The misalignment in the $y$-direction for the entry of the beam element (default: 0 m ). DY $>0$ displaces the element in the positive $y$-direction (see Figure 8.3).

DS
The misalignment in the $s$-direction for the entry of the beam element (default: 0 m ). DS $>0$ displaces the element in the positive $s$-direction (see Figure 8.1).

DPHI The rotation around the $x$-axis. A positive angle gives a greater $y$-coordinate for the exit than for the entry (default: 0 rad, see Figure 8.3).

DTHETA The rotation around the $y$-axis according to the right hand rule (default: 0 rad, see Figure 8.1).

DPSI The rotation around the $s$-axis according to the right hand rule (default: 0 rad, see Figure 8.1).

MREX The horizontal read error for a monitor. This is ignored if the element is not a monitor (see Figure 8.4). If MREX $>0$, the reading for $x$ is too high (default: 0 m ).

MREY The vertical read error for a monitor. This is ignored if the element is not a monitor (see Figure 8.4). If MREY $>0$, the reading for $y$ is too high (default: 0 m ).
Example:
EALIGN, QF [2], $\mathrm{DX}=0.002, \mathrm{DY}=0.0004 * \operatorname{RANF}(), \mathrm{DPHI}=0.0002 * \operatorname{GAUSS}()$
Refer to Section 2.4.8 for random value formats.


Figure 8.1: Example of Misplacement in the ( $x, s$ )-plane

### 8.2 Field Error Definitions

Field errors can be entered as relative or absolute errors. Different multipole components can be specified with different kinds of errors (relative or absolute). If an attempt is made to assign both a relative and an absolute error to the same multipole component, the absolute error is used and a warning is given. Relative errors cannot be assigned to an element of the type MULTIPOLE. Relations between absolute and relative field errors are listed below.

All field errors are specified as the integrated value $\int K_{i} d s$ along the magnet axis in $\mathrm{m}^{-i}$ (see Section 1.4). At present field errors may only affect field components allowed as normal components in a magnet. This means for example that a dipole may have errors of the type dipole, quadrupole, sextupole, and octupole; but not of the type decapole. There is no provision to specify a global relative excitation error affecting all field components in a combined function magnet. Such an error may only be entered by defining the same relative error for all field components.

Field errors can be specified for all magnetic elements by one of the statements


Figure 8.2: Example of Misplacement in the $(x, y)$-plane


Figure 8.3: Example of Misplacement in the $(y, s)$-plane

```
EFIELD, RANGE=range,TYPE=name,RADIUS=real,DBL=real,DKL(i)=real,&
    DBLR=real,DKLR(i)=real,BROT=real,ROT(i)=real
EFCOMP, RANGE=range,TYPE=name,DBLN=real,DKLN(i)=real,&
    DBLS=real,DKLS(i)=real
```

Each new EFIELD or EFCOMP statement replaces the field errors for all elements in its range (s). Any old field errors present in the range are discarded. EFIELD defines the error in terms of relative or absolute amplitude and rotation; while EFCOMP defines them in terms of absolute components.

The meaning of the attributes is:
RANGE A description of the beam elements in which the error occurs (see Section 4.7). It can contain up to five different ranges:
EFIELD , QF , QD [2]/QD [4] ,\#3/\#5,\#13/\#15

TYPE Label(s) attached to physical element(s) by a TYPE=name clause. May contain up to five different types like

```
EFIELD,TYPE=MQ,MA ,MB,MC,MD
```

Radius $R$ were $\operatorname{DKLR}(i), 1 \leq i \leq 10$ is specified (default 1 m ). This attribute is required if DBLR or any DKLR(i) is specified.

DBL Absolute error amplitude for the dipole strength (default: 0 rad ). Its value is $\int K_{0} d s$ over the length of the magnet (see below).


Figure 8.4: Example of Read Errors in a monitor

DKL (i) Absolute error amplitude for the multipole strength with ( $2 * i+2$ ) poles (default: $0 \mathrm{~m}^{-n}$ ) Its value is $\int K_{i} d s$ over the length of the magnet (see below).

DBLR Relative error in the dipole strength (default: 0 , see below).
$\operatorname{DKLR}(\mathrm{i}) \quad$ Relative error in multipole strength with $(2 * i+2)$ poles (default: 0 , see below). This attribute requires that RADIUS is also given.

BROT Rotation angle $\phi_{0}$ for the dipole error (default: 0 , see Figure 8.2).
ROT (i) $\quad$ Rotation angle $\phi_{i}$ for the multipole with $(2 * i+2)$ poles, where $1 \leq i \leq 10$ (default: 0 rad, see Figure 8.2).

DBLN Absolute error for the horizontal dipole strength (default: 0 ).
$\operatorname{DKLN}(i) \quad$ Absolute error for the normal multipole strength with $(2 * i+2)$ poles (default: 0 ).
DBLS
Absolute error for the vertical dipole strength (default: 0 ).
DKLS (i) Absolute error in skewed multipole strength with $(2 * i+2)$ poles (default: 0 ).
The dimensions for $\operatorname{DKL}()$, $\operatorname{DKLR}()$, $\operatorname{DKLN}()$, $\operatorname{DKLS}()$, and ROT() are preset to 10 in the command dictionary, but this can be changed easily. Examples:

```
EFIELD,TYPE=MQ,DBL=5.0E-4,DKL(3)=0.0025*RANF(),DKL(5)=0.0092*GAUSS()
EFIELD,TYPE=MQ,DBLN=5.0E-4,DKLN(3)=0.0025*RANF(),DKLN(5)=0.0092*GAUSS()
```

Refer to Section 2.4.8 for random value formats.

### 8.2.1 Field Errors in Bending Magnets

Only the normal dipole, quadrupole, and sextupole components of the error field are considered in a bending magnet. They are rotated together with the main field. The error field components have the values:

$$
\begin{aligned}
& \Delta K_{0} L=\operatorname{DBL}=\operatorname{DBLN}=K_{0} L \cdot \operatorname{DBLR}, \\
& \Delta K_{1} L=\operatorname{DKL}(1) \cos \phi_{1}=\operatorname{DKLN}(1)=K_{0} L \frac{1!}{R} \cdot \operatorname{DKLR}(1) \cdot \cos \phi_{1}, \\
& \Delta K_{2} L=\operatorname{DKL}(2) \cos \phi_{2}=\operatorname{DKLN}(2)=K_{0} L \frac{2!}{R^{2}} \cdot \operatorname{DKLR}(2) \cdot \cos \phi_{2}, \\
& \Delta K_{3} L=\operatorname{DKL}(3) \cos \phi_{3}=\operatorname{DKLN}(3)=K_{0} L \frac{3!}{R^{3}} \cdot \operatorname{DKLR}(3) \cdot \cos \phi_{3} .
\end{aligned}
$$

### 8.2.2 Field Errors in Quadrupoles

Only the normal quadrupole component of the error field, that is the excitation error, is considered. It is rotated together with the main field. The error field component has the value:

$$
\Delta K_{1}=\operatorname{DKL}(1) \cdot \cos \phi_{1}=\operatorname{DKLN}(1)=K_{1} L \cdot \operatorname{DKLR}(1) \cdot \cos \phi_{1} .
$$

### 8.2.3 Field Errors in Sextupoles

Only the normal sextupole component of the error field, that is the excitation error, is considered. It is rotated together with the main field. The error field component has the value:

$$
\Delta K_{2} L=\operatorname{DKL}(2) \cdot \cos \phi_{2}=\operatorname{DKLN}(2)=K_{2} L \cdot \operatorname{DKLR}(2) \cdot \cos \phi_{2} .
$$

### 8.2.4 Field Errors in Multipoles

Error fields and main field are rotated separately and then added. The error field components are always absolute values:

$$
\begin{aligned}
& \left(\Delta B_{x 0}+i \cdot \Delta B_{y 0}\right) L=\operatorname{DBL} \cdot \exp (-i \cdot \operatorname{BROT})=\operatorname{DBLN}+i \cdot \operatorname{DBLS}, \\
& \left(\Delta K_{x k}+i \cdot \Delta B_{y k}\right) L=\operatorname{DKL}(k) \cdot \exp (-(k+1) i \cdot \operatorname{ROT}(k))=\operatorname{DKLN}(k)+i \cdot \operatorname{DKLS}(k) .
\end{aligned}
$$

### 8.2.5 Field Errors in Orbit Correctors

The dipole error field is added to the main field and then rotated. A rotation (TILT) must not be used when closed orbit corrections are to be computed. The error field components are always absolute values:

$$
\left(\Delta B_{0}+i \cdot \Delta B_{y 0}\right) L=\text { DBL } \exp (-i \cdot \operatorname{BROT})=\text { DBLIN }+i \cdot \text { DBLS } .
$$

### 8.3 Error Option Command

EOPT The random generator for MAD is taken from [21]. The error option command specifies different seeds for random values:

```
EOPT,SEED=integer
```

SEED Selects a particular sequence of random values. A SEED value is an integer in the range [ $0 \cdots 999999999$ ] (default: 123456789). SEED alone continues with the current sequence (see also Section 2.4.5). on top of existing ones. If it is not set, new errors overwrite any previous definitions.

Example:

### 8.4 Error Print Command

This command prints a table of errors assigned to elements. The range for these elements has to be specified. Field errors are printed as absolute errors, because all relative errors are transformed to the corresponding absolute error at definition time. An error print is requested by the EPRINT statement

```
EPRINT,RANGE=range{,range},TYPE=name{,name},FULL
```

It has three attribute:
RANGE Up to five ranges for printing (see Section 4.7).
TYPE Up to five element types for printing.
FULL If this logical flag is true, all positions are printed.
Example:

```
EPRINT,QD[1]/QD[5]
```

The selection of positions is done as for PRINT (see Section 5.3).

## Chapter 9. Closed Orbit Commands

Table 9.1: Commands Related to the Closed Orbit

| Name | Function | Section |
| :--- | :--- | :--- |
| CORRECT | Complete correction algorithm | 9.6 |
| GETDISP | Read table of dispersion readings | 9.1 |
| GETKICK | Read table of corrector settings | 9.2 |
| GETORBIT | Read table of monitor readings | 9.3 |
| MICADO | Correction by MICADO algorithm | 9.5 |
| PUTDISP | Write table of dispersion readings | 9.1 |
| PUTKICK | Write table of corrector settings | 9.2 |
| PUTORBIT | Write table of monitor readings | 9.3 |
| USEKICK | Activate/deactivate Correctors | 9.4 |
| USEMONITOR | Activate/deactivate Monitors | 9.4 |

There are six commands related to the closed orbit, listed in Table 9.1. Before using one of them a beam line must be selected by means of a USE command (see Section 5.1).

### 9.1 GETDISP and PUTDISP Statements

The two commands

```
GETDISP,FILENAME=file-name
PUTDISP,FILENAME=file-name
```

serve the following purposes:
GETDISP Reads a TFS file file-name containing dispersion readings for the orbit position monitors (see Appendix C). This file may have been written by a PUTDISP command or by an external program. The readings can be used in a subsequent MICADO command to find corrector settings for correcting the vertical dispersion.

PUTDISP Finds the closed orbit and writes the dispersion readings at all orbit position monitors on a TFS file file-name. (see Appendix C). This file may be read again by MAD or an external program.

The format of file-name is described in Appendix B. The default file-name is dispersion. The table must have the following columns:

PUNAME $\quad$ Name of the monitor (string).
DX Horizontal dispersion (real).
DY Vertical dispersion (real).
STATUS Integer zero for active, nonzero for inactive.

### 9.2 GETKICK and PUTKICK Statements

The two commands

```
GETKICK,FILENAME=file-name [,ADD]
PUTKICK,FILENAME=file-name [,PLANE=X|Y]
```

serve the following purposes:
GETKICK Reads a TFS file file-name containing the settings for the orbit correctors (see Appendix C). This file may have been written by a PUTKICK command or by an external program. The excitations will be used for any subsequent closed orbit search, unless changed by MICADO or CORRECT commands. If the flag ADD is true, the setting read are added to any previous settings.

PUTKICK Writes the current setting of the orbit correctors on a TFS file file-name. (see Appendix C). This file may be read again by MAD or an external program.

The format of file-name is described in Appendix B. The default file-name is setting. For reading the table must have the following columns:

STR_NAME Name of the corrector (string).
K_N_H Horizontal setting (real).
K_N_V Vertical setting (real).
STATUS Integer zero for active, nonzero for inactive.
By default the table written has the same format. If the attribute PLANE=X or PLANE=Y is entered, only the strengths for that plane are written, and there is only one column for settings, headed by K.N. Such a table cannot be read by MAD.

### 9.3 GETORBIT and PUTORBIT Statements

The two commands

```
GETORBIT,FILENAME=file-name
PUTORBIT,FILENAME=file-name
```

serve the following purposes:
GETORBIT Reads a TFS file file-name containing orbit readings for the orbit position monitors (see Appendix C). This file may have been written by a PUTORBIT command or by an external program. The readings can be used in a subsequent MICADO command to find corrector settings for correcting the orbit positions.

PUTORBIT Finds the closed orbit and writes the readings of all orbit position monitors on a TFS file file-name (see Appendix C). This file may be read again by MAD or an external program.

The format of file-name is described in Appendix B. The default file-name is orbit. The table must have the following columns:

PUNAME Name of the monitor (string).
$\mathrm{X} \quad$ Horizontal position (real).
Y Vertical position (real).
STATUS Integer zero for active, nonzero for inactive.

### 9.4 USEKICK and USEMONITOR, Activate/deactivate elements

To provide more flexibility with orbit and dispersion correction two commands are provided:

```
USEMONITOR, STATUS=flag,MODE=name,RANGE={range}
USEKICK, STATUS=flag,MODE=name,RANGE={range}
```

The purpose of the two commands is:
USEMONITOR Activates or deactivates a selection of beam position monitors. This command affects elements of types MONITOR, HMONITOR, of VMONITOR.

USEKICK Activates or deactivates a selection of orbit correctors. This command affects elements of types KICKER, HKICKER, of VKICKER.

Both commands have the same attributes:
STATUS If this flag is true (on), the selected elements are activated. Active orbit monitor readings will be considered, and active correctors will be allowed to vary in subsequent correction commands. Inactive elements will be ignored subsequently.

MODE This name may take the value ALL, in which case all correctors or monitors will be set to active or inactive. For the USEKICK command two more values are possible:

USED Changes all correctors which have been set to a non-zero value by a preceding correction.
UNUSED Changes all correctors which have never been set by a preceding correction.

RANGE This attribute is only considered if MODE is not given. Up to five ranges may be specified to select correctors or monitors to be affected.

One may do two iterations with disjoint sets of correctors as follows:

```
USE,... ! set working beam line
...
CORRECT,NCORR=32
USEKICK, OFF,USED
CORRECT,NCORR=32
```

```
| define imperfections
```

| define imperfections
| use 32 correctors
| use 32 correctors
! deactivate used correctors
! deactivate used correctors
! uses different set of correctors

```
! uses different set of correctors
```

Forcing reuse of the same set is achieved thus:

```
USE,... ! set working beam line
... ! define imperfections
CORRECT,NCORR=32 ! use 32 correctors
USEKICK,OFF,UNUSED ! deactivate unused correctors
CORRECT,NCORR=32 ! uses same set of correctors
```


### 9.5 MICADO Statement

The command MICADO assumes that the closed orbit of the machine is known, and that the table of monitor readings (and optionally of dispersion readings) exists. These can be the result of previous GETORBIT and GETDISP commands.

```
MICADO, ERROR=real,NCORR=integer [,C2LIST] [,M1LIST]
    [,DXWEIGHT=real] [,DXWEIGHT=real] [PLANE=letter]
```

It causes closed orbit correction by the MICADO method [2]. The program assumes that all correctors are available for use, but it limits its choice to the most effective NCORR correctors. The monitors are not used for orbit or dispersion readings, if flagged as inactive in the corresponding TFS files read. The attributes have the following meaning:

ERROR The desired accuracy of the correction (r.m.s. error of the closed orbit, default: 0 m ).
NCORR The maximum number of correctors to be used (correctors are selected by the program, default: all).

DXWEIGHT The weight for horizontal dispersion correction. At present this should be left at zero to omit dispersion correction in the horizontal plane.

DYWEIGHT The weight for vertical dispersion correction. If this weight is non-zero, an attempt is made to correct the vertical dispersion.

PLANE If this attribute is X , only the horizontal correction is made; it it is Y , only the vertical correction is made. In all other cases both planes are corrected.

Two attributes affect the printing of tables:
C2LIST Corrector settings after correction,
M1LIST Monitor readings before correction,
Both have the same possible values:
NONE Print summary only for this table (default),
USED Print table of all used correctors or monitors, plus summary,
ALL Print table of all correctors or monitors, plus summary.
Example:

```
MICADO,ERROR=1.E-4,C2LIST,M1LIST
```


### 9.6 CORRECT Statement

The CORRECT statement makes a complete closed orbit correction by the MICADO method, using the computed values at the monitors:

```
CORRECT, NCORR=integer,ERROR=value,ITERATE=integer,&
    C1LIST,C2LIST,M1LIST,M2LIST&
    [,DXWEIGHT=real] [,DYWEIGHT=real] [,PLANE=letter]
```

The program always assumes that all correctors and monitors are available for use, but at each iteration it limits its choice to NCORR most effective correctors. The attributes have the following meaning:

NCORR The maximum number of correctors to be used (correctors are selected by the program, default: all). At each iteration the program may select a different set of NCORR correctors, thus using more than NCORR correctors in total.

ERROR The desired accuracy of the correction (r.m.s. error of the closed orbit, default: 0 m ).

ITERATE The number of iterations to be made on the non-linear problem (default: 1). If ITERATE $=0$, the monitor and corrector tables are printed, but no correction is made.

DXWEIGHT The weight for horizontal dispersion correction. At present this should be left at zero to omit dispersion correction in the horizontal plane.

DYWEIGHT The weight for vertical dispersion correction. If this weight is non-zero, an attempt is made to correct the vertical dispersion.

PLANE If this attribute is X , only the horizontal correction is made; it it is Y , only the vertical correction is made. In all other cases both planes are corrected.

Four attributes affect the printing of tables:
C1LIST Corrector settings before correction,
C2LIST Corrector settings after correction,
M1LIST Monitor readings before correction,
M2LIST Monitor readings after correction.
All four have the same possible values:
NONE Print summary only for this table (default),
USED Print table of all used correctors or monitors, plus summary,
ALL Print table of all correctors or monitors, plus summary.
Example:

```
CORRECT,ERROR=1.E-4,NCORR=100,ITERATE=3,C2LIST,M1LIST
```


# Chapter 10. Plotting and Tabulating 

Table 10.1: Plotting Commands

| Name | Function | Section |
| :--- | :--- | :--- |
| SETPLOT | Set plot options | 10.1 |
| RESPLOT | Set default plot options | 10.1 |
| PLOT | Build a plot | 10.2 |
| STRING | Define a table expression | 10.5 |
| TABLE | Print selected columns of a table | 10.5 |

### 10.1 SETPLOT and RESPLOT Commands

The SETPLOT command allows to specify parameters common to all subsequent plots:

```
SETPLOT, FONT=integer,LWIDTH=real,XSIZE=real,YSIZE=real,&
    ASCALE=real,LSCALE=real,SSCALE=real,TSCALE=real
```

The RESPLOT command serves to reinstall the defaults for the SETPLOT command parameters:

## RESPLOT

It is defined in the command dictionary as

```
RESPLOT: SETPLOT,FONT=1,LWIDTH=1.,XSIZE=0.,YSIZE=0.,&
ASCALE=1., LSCALE=1., SSCALE=1.,TSCALE=1.
```

To change the plot size, a SETPLOT command specifying both XSIZE and YSIZE must be given before the first PLOT command. These parameters must both be entered to be effective. They allow to make long plots on the Versatec plotter. The values ( $0 ., 0$.) give the default on any device. See GXPLOT manual for details [15]. They may be set to almost the page size for A4 format output, i. e. to 20 and 28 cm .

XSIZE Horizontal size of plot on the hard copy output device [cm],
YSIZE Vertical size of plot on the hard copy output device [cm].
All other SETPLOT options become effective when set, and remain so until redefined.
FONT Character font, (default: 1, Roman). Other fonts exist, but are normally specific to the GKS-package used (see Appendix of the GKSGRAL manual [1]), and device-dependent.

LWIDTH A line width scale factor applying to curves (device-dependent). Accepted values are normally 1,3 , and 5 (default: 1 ).

ASCALE Scale factor for the size of the plotted characters of the curve annotation,
LSCALE $\quad$ Scale factor for the size of the plotted characters of the axis labels (scale),
SSCALE Scale factor for the size of the plotted symbol at the point positions,
RSCALE $\quad$ Scale factor for the size of the plotted characters of the axis text.
The default is 1.0 for all four scale factors.

### 10.2 PLOT Command

The PLOT command produces one or several frames (pictures) at a time:

```
PLOT VAXIS=name,VAXIS1=name,VAXIS2=name,VAXIS3=name,VAXIS4=name,&
    HAXIS=name,BARS=integer,STYLE=integer,SYMBOL=integer,&
    MAXPLOT=integer,SORT=logical,SPLINE=logical,MULTIPLE=logical,&
    FFT=logical,HMIN=real,HMAX=real,VMIN=real,VMAX=real,&
    TABLE=name,TITLE=string,PARAM=name,RANGE=range,DELTAP=real,&
    PARTICLE=integer,TURNS=integer
```

Its parameters are only valid for this one command:
VAXIS Up to five names of dependent (vertical) variables to be plotted on the same axis (!). If more than one vertical axis is required, one may enter up to five variable names per axis via one of the parameters VAXIS1, VAXIS2, VAXIS3, VAXIS4. If VAXIS is specified, the others are ignored. Example:

```
PLOT,HAXIS=S,VAXIS=BETX,BETY
```

gives a plot with BETX and BETY both referring to the same axis, whereas
PLOT, HAXIS=S, VAXIS1=BETX,VAXIS2=BETY
provides separate axes (with different scales, normally) for the two variables.
HAXIS The name of the independent (horizontal) variable.
BARS Connect each point to the horizontal axis by a vertical line.
STYLE Line style selection for all curves. The default value is 1 (solid line for all curves). Further possible values are: 0 (no connecting line, i.e. nothing at all if no symbol plotted, see below), 2 (dashed), 3 (dotted), 4 (dot-dashed), and others mentioned in the GKSGRAL manual. A special value is 100 : when chosen, it provides line styles 1,2 , $3,4,1,2$, etc. for successive curves in the same frame.

SYMBOL This option will be mainly used for track plots, and allows to plot symbols at the points which are plotted. These can be one of the following: "hardware" symbols, selected by the values 1 (dot), $2(+), 3\left(^{*}\right), 4(o)$, or $5(x)$; the turn number (curve number for tables TWISS and TUNES) in each picture, selected by value 100; an upper case character A...Z (201 to 226), lower case character ( 301 to 326 ), or a digit $0 . . .9$, selected by 400 to 409 . 200 and 300 act similar to 100: 200 makes turns (curves) loop over A...Z, 300 over a...z. The SSCALE factor on the SETPLOT command applies to all of these except the "hardware" symbols which are of fixed size.

MAXPLOT Defines the maximum number of pictures (frames) to be plotted with one command (default: 10). This is a protection against user mistakes mainly (e.g. plotting one frame per $s$ value around the whole machine).

SORT Logical flag, sorts the points in ascending values of the abscissa.
SPLINE Logical flag, connects curve points by a third order natural spline rather than by straight line segments (for $\alpha, \beta, \mu$ and $D$ the correct formulae are used for interpolation if plotted against $s$ ).

MULTIPLE Logical flag, only for TRACK table plots: put all particles into one frame.

Plot the result of the Fourier analysis of the picture, rather than the picture itself (not yet implemented).

TABLE Name of the table to be plotted. If omitted, the last generated table will be used.
TITLE Text string to be put on top of the picture (default: run title as read on TITLE command).

PARAM Frame selection parameter: S or DELTAP for the TWISS table, PARTICLE or TURNS for the TRACK table.

RANGE Machine range to be used (see Section 4.7).
DELTAP Values of DELTAP to be used (as on TWISS command).
PARTICLE Particle number.
TURNS Turn number.
The axis ranges are specified by lower and upper values. If neither value is given, scaling is fully automatic, if both are given, they are used (possibly rounded). If only the lower limit is given as zero, and all corresponding curves lie entirely above or below zero, the scale starts or ends at zero. If only the upper limit is given as zero, the scale is symmetric to zero.

HMIN Lower limit of the abscissa.
HMAX Upper limit of the abscissa.
VMIN Up to four lower limits for each of the four vertical axes.
VMAX Up to four upper limits for each of the four vertical axes.

### 10.3 Axis and frame selection

All variables from the TWISS table, the TUNES table, and the TRACK table can be plotted on the horizontal or vertical axis. For a list, see Sections 1.5, 1.5.5, and 1.5.6. In addition, the variables RBETX, RBETY, RBXMAX, and RBYMAX can be requested. They stand for the square root of the variables behind the ' R '.

The variables $S$ and DELTAP in the TWISS table, and the variables PARTICLE and TURNS in the TRACK table are used as frame parameters PARAM, either automatically or user-driven. This means that for each value of this variable a new picture is plotted, as described below for the TWISS table.

### 10.3.1 TWISS table

Depending on the user specification when calculating the table this will normally contain Twiss parameter values for a number of $S$ values (around the ring), and for one or more DELTAP values (see Section 1.5). The table may therefore be regarded as a "matrix" with $S$ as row index, DELTAP as column index, and each "element" consisting of all Twiss values for a given pair (S, DELTAP), both S and DELTAP included in these values.

A subset of the table can be chosen by specifying a RANGE for $S$, and a list of values for DELTAP. If no $S$ range is given, all $S$ values are selected (!), and similarly for DELTAP: if no values given, all values are used.

Out of this (sub-)matrix the user can now select variables to be plotted. Let us suppose that the user always chooses BETY as ordinate. If S is abscissa, BETY will be plotted against $S$ with DELTAP as "frame parameter" (PARAM), i.e. for each selected value of DELTAP there will be a new plot frame.

Similarly for DELTAP as abscissa: in this case, BETY will be plotted against DELTAP for each valid S, giving a new plot frame each time (this might be quite a lot if the user makes a mistake; in order to avoid excessive output, the number of frames per plot command will be limited, with an option MAXPLOT to explicitly changing this limit).

In the case that neither S nor DELTAP is horizontal axis (e.g. plotting BETY against BETX, whatever this may yield), the user has to specify in addition what his frame parameter is. For each value of this frame parameter ( $S$ or DELTAP) the values in the corresponding row or column will be plotted in a separate frame. The default is DELTAP. The value of PARAM is ignored if HAXIS=S or DELTAP, i.e. the "other" one is chosen automatically (if HAXIS=DELTAP, then $S$ will act as frame parameter).

Examples:

```
PLOT, HAXIS=S,VAXIS=BETY
```

produces $n$ plots (for the $n$ DELTAP values in the table) with one curve each showing the dependence of BETY on $S$.

```
PLOT,HAXIS=S,VAXIS=BETY,DELTAP=0.:0.01:0.01
```

produces two plots of this type, for $\operatorname{DELTAP}=0$. and 0.01 (provided the DELTAP values exist in the table).

```
PLOT, HAXIS=DELTAP,VAXIS=BETY,RANGE=#E
```

produces one plot BETY against DELTAP at the position of the last element in the table.
PLOT , HAXIS=DELTAP, VAXIS=BETY
will produce 10 plots (default value of MAXPLOT) for the first 10 S values in the table.

```
PLOT,HAXIS=BETX,VAXIS=BETY,PARAM=DELTAP,DELTAP=0.
```

will produce one plot (at DELTAP $=0$ ), the curve points being (BETX,BETY) at all $S$ values. Here, PARAM = DELTAP may be omitted since this is the default.

PLOT, HAXIS=BETX,VAXIS=BETY, PARAM=S, RANGE=\#E
will produce one plot at the last $S$ value, with (BETX, BETY) points for all DELTAP values in the table.

### 10.3.2 TRACK table

The TRACK table (see 1.5.6) has the same structure as the TWISS table, with PARTICLE as row index, and TURNS as column index each of which can therefore be chosen as frame selection parameter. The default parameter is PARTICLE. The value of PARAM is ignored if HAXIS=PARTICLE or TURNS, i.e. the "other" one is chosen automatically (if HAXIS=TURNS, then PARTICLE will act as frame parameter). The option MULTIPLE allows to plot the points of all particles in the same frame.

```
PLOT,HAXIS=X,VAXIS=PX,PARTICLE=1,3,5,MULTIPLE
```

will produce one plot of PX versus X for particles 1,3 , and 5 .

### 10.3.3 TUNES table

The TUNES table (see Section 1.5.5) does not have frame-parameters. If PARAM is specified, it will be ignored.

```
PLOT,TABLE=TUNES, HAXIS=DELTAP,VAXIS=QX
```

will produce a plot of the x -tune versus DELTAP.

### 10.4 Tune Grid

MAD permits to plot an tune grid if the following two conditions are both true:

- A plot of $Q_{x}$ versus $Q_{y}$ or of $Q_{y}$ versus $Q_{x}$ is requested,
- At least one selection criterion has been entered on the PLOT command.

MAD then plots all resonance lines that are inside the plot window, and which fulfil conditions of the form

$$
k_{x} Q_{x}+k_{y} Q_{y}+k_{s} Q_{s}=p s,
$$

where $k_{x}, k_{y}, k_{s}$, and $p$ are integers, and $s$ is the super-periodicity (integer). Furthermore, the user must impose at least one condition of the form

$$
F\left(k_{x}, k_{y}, k_{s}\right)=n_{1}: n_{2}: n_{3},
$$

where $F$ is an arithmetic function $k_{x}, k_{y}, k_{s}$, and integers only, and only the operators "+", "-", "*", and "/" are allowed. $n_{1}, n_{2}$, and $n_{3}$ are literal integers, and $n_{1}: n_{2}: n_{3}$ stands for a range. It is interpreted like the range in a FORTRAN DO loop, i. e. it runs through the values $n_{1}, n_{1}+n_{2}, \ldots, n_{3}$.

Examples:

$$
k_{s}=-2: 2: 2 \Rightarrow k_{s}=-2,0,+2 ; \quad k_{x}+k_{y}=0: 0: 1 \Rightarrow k_{x}=-k_{y}
$$

Note that integer arithmetic is used, therefore the condition

$$
2 *\left(k_{x}+k_{y} / 2\right) *\left(k_{s}-1\right) / 2=-3: 3: 2
$$

will do something weird.
Criteria are specified via the following new attributes of the PLOT command:

```
PLOT, . . , NTMAX=integer,QCONDi=string,integer1,integer2,integer3
```

with the meaning:
NTMAX Maximum for $\left|k_{x}\right|+\left|k z_{y}\right|+\left|k_{s}\right|$ (default: 20). The absolute upper limits are $\left|k_{x}\right|+\left|k_{y}\right|+$ $\left|k_{s}\right| \leq 20$, and $\left|k_{s}\right| \leq 10$.

QCONDi The function $F$, encoded as a quoted string, and the three integers $n_{1}, n_{2}, n_{3}$ for the selection criterion (defaults: " ", 0, 0, 1). The digit i may run from 1 to 10 for up to 10 conditions. When several selection criteria are entered, they must all be true for a line to be plotted.
Example:

```
PLOT,HAXIS=QX,VAXIS=QY,&
    HMIN=21., HMAX=23.,VMIN=18., VMAX=20.,&
    NTMAX=5,QS=0.2,&
    QCOND1="KX",1,1,&
    QCOND2="KS",-3,3,&
    QCOND3="KX+KY",-10,10,10
```

will plot $Q_{y}$ versus $Q_{x}$, and superpose the lines

$$
\begin{aligned}
& k_{x} Q_{x}=p s, k_{x}=1 ; \quad k_{s} Q_{s}=p s, k_{s}=-3,-2,-2,0,1,2,3 \\
& k_{x} Q_{x}+k_{y} Q_{y}=p s, k_{x}+k_{y}=-10,0,10
\end{aligned}
$$

The line style reflects the importance of the resonance. The thickness varies from $\left|k_{x}\right|+\left|k_{y}\right|=1$ (thickest) to $\left|k_{x}\right|+\left|k_{y}\right|=5$ (thinnest) and remains constant from there on. Lines with $k_{s}=0$ are solid, with $\left|k_{s}\right|=1$ dashed, with $\left|k_{s}\right|=2$ dot-dashed, and with $\left|k_{s}\right|>2$ dotted.

### 10.5 TABLE and STRING Commands

Any table created by MAD can be tabulated on the PRINT file by the command

$$
\text { TABLE , NAME=name, COLUMNS=name }\{, \text { name }\}, S U M=\text { name }\{, \text { name }\}
$$

It has the attributes
NAME Name of the table to be tabulated. By default the last table created is used.
COLUMNS Up to 50 names whose values shall be tabulated. These may be any of

- Name of a column in the table to be listed.
- Name of a descriptor in the table to be listed.
- Name of a global variable.
- Name of a global STRING expression (see below).
- Name of an expression predefined for the table.

SUM Up to 50 name of columns to be summed. The TABLE command will compute (but not print) and sum up each name for each table row, and print the total after the table. This may be used to compute approximate values for synchrotron integrals.

A STRING expression is defined as follows:

```
label:STRING,"string"
```

where label is the name given to the expression, and string (enclosed in single or double quotes) must contain a valid expression. Operands in such an expression may include:

- Name of a column in the table to be listed.
- Name of a descriptor in the table to be listed.
- Name of a global variable.
- Name of an attribute of a command or definition.

A string may be redefined by repeating the STRING command.
Example:

```
OPTICS,...
SIGX:STRING,"SQRT(BEAM[EX]*BETX)"
CHROMX:STRING,"SQRT(BETX*BEAM[EX] + (DX*DELTAP)^2)
TABLE,NAME=OPTICS,COLUMN=K1L,K2L,BETX,SIGX,SUM=CHROMX
```


### 10.5.1 Plotting Composite Values

The mechanisms from the preceding subsection can also be used for plotting. One may for example plot the beam size after a NORMAL command by

```
NORMAL,...
SIGX:STRING,&
"SQRT(BEAM[EX]*(E11^2+E21^2) + BEAM[EY]*E31^2+E42^2) + BEAM[ET]*(E51^2+E61^2))"
SIGY:STRING,&
"SQRT(BEAM[EX]*(E13^2+E23^2) + BEAM[EY]*E33^2+E43^2) + BEAM[ET]*(E53^2+E63^2))"
SIGT:STRING,&
"SQRT(BEAM[EX]*(E15^2+E25^2)+ BEAM[EY]*E35^2+E45^2)+ BEAM[ET]*(E55^2+E65^2))"
PLOT,TABLE=EIGEN,HAXIS=S,VAXIS=SIGX,SIGY,SIGT
```

If desired, such expressions can be predefined and attached to a given table type. Such predefinitions would be placed in the MAD command dictionary:

```
T_TWISS:KEYWORD, \(\mathrm{PR}=3, \mathrm{SP}=2, \&\)
    GAMX=(S(3),=3\*""), GAMY=(S(3),=3\*""),\&
    SIGX=(S(3),=3\*""),SIGY=(S(3),=3\*"")
T_TWISS:T_TWISS,\&
    GAMX=" (1+ALFX^2)/BETX", GAMY=" (1+ALFY^2)/BETY",\&
    SIGX="SQRT (BETX*BEAM[EX])",SIGY="SQRT (BETY*BEAM[EY])"
```

The name of the definition must be the table type, prefixed by $T_{-}$. Note that the strings in the first definition must be dimensioned at 3 , thus allowing to enter information like dimension and axis label information to the second definition.

## Chapter 11. HARMON Module

Table 11.1: HARMON Commands

| Name | Function | Section |
| :--- | :--- | :--- |
| HARMON | Set up HARMON tables | 11.1 |
| HCELL | Minimise aberrations | 11.4 |
| HCHROMATICITY | Calculate chromaticity | 11.2 |
| HFUNCTION | Calculate harmonic functions | 11.4 |
| HRESONANCE | Calculate resonance effects | 11.3 |
| HTUNE | Tune chromaticity | 11.2 |
| HWEIGHT | Set matching weights for resonances | 11.4 |
| HVARY | Make a multipole variable | 11.4 |
| ENDHARM | Quit HARMON program | 11.1 |

The harmon program by M. H. Donald [12] can be called directly from MAD. ${ }^{1}$ Before calling harmon, a beam line must be selected by means of a USE command (see Section 5.1). The HARMON command sets up internal tables and defines the emittances to be used for the analysis. The internal tables are released and HARMON mode is terminated by the ENDHARM command. The available commands are listed in Table 11.1.

It is important to note that HARMON does not consider the following:

- Coupling effects,
- Excitation errors (EFIELD, EFCOMP),
- Alignment errors (EALIGN).

The integral formalism used does not allow to take these effects into account, and users should be aware of this.

### 11.1 Activating and Deactivating HARMON

The HARMON module is activated and the relevant tables are initialised by the command

```
HARMON,FX=real,FY=real,FE=real
```

with the attributes:
FX Number of horizontal standard deviations to be taken,
FY Number of vertical standard deviations to be taken,
FE Number of longitudinal standard deviations to be taken.
If one of these values is omitted or 0 , the program uses 1 . The previous attribute OBSERVE no longer exists. If the machine has a symmetry, a second observation point is given at the symmetry point.

The HARMON module is deactivated and its tables are released by the ENDHARM command

> ENDHARM

An example is given at the end of this Chapter.

[^2]
### 11.2 Chromaticity Calculation or Tuning

The chromaticity is calculated by a command without attributes:

```
HCHROMATICITY
```

The chromaticity is tuned to a desired value by the HTUNE command

```
HTUNE,QX'=real, QY'=real,TOLERANCE=real
```

The desired values are entered as $d Q / d \delta$ with the attributes
QX $\quad$ Desired horizontal chromaticity (default: 0 ),
QY $\quad$ Desired vertical chromaticity (default: 0 ).
TOLERANCE Tolerance desired (default: 0 , i.e. as good as feasible).
The multipoles to be varied are specified with a HVARY command (see Section 11.4). Example:

```
HVARY,SF[K2] ,STEP=0.001
HVARY,SD[K2],STEP=0.001
HVARY,SFK2,STEP=0.0001
HTUNE,QX'=0.0,QY'=0.0
```


### 11.3 Resonance Calculations

The resonance effects are computed by the HRESONANCE command
HRESONANCE, ORDER=integer, DISP=logical

The order of the resonance is selected by the attribute
ORDER Order of the resonance. This causes computation of the resonances

$$
N_{1} Q_{x}+N_{2} Q_{y}=p_{1}, \quad N_{1} Q_{x}-N_{2} Q_{y}=p_{2},
$$

with integer $p_{1}, p_{2}$, and the condition $N_{1}+N_{2}=O R D E R$,
DISP If set, the dispersion is included in the computation.
Contrary to previous versions of HARMON, all sextupole components are considered, i.e. all elements of the types RBEND, SBEND, SEXTUPOLE, MULTIPOLE. Example:

```
HRESONANCE,ORDER=3,DISP
```


### 11.4 Distortion Functions

The HVARY command makes a multipole component variable:
HVARY, NAME=variable, STEP=real, LOWER=real, UPPER=real

The command has the following attributes:
NAME The name of a variable to be varied. It must be the sextupole, octupole or decapole component of an element having one of the types SBEND, RBEND, SEXTUPOLE, or MULTIPOLE.

STEP An initial step size to be used in matching (required),
LOWER Lower limit for the multipole strength,
UPPER Upper limit for the multipole strength.
The units for STEP, LOWER, and UPPER are the same as in the corresponding element definition. Examples:

```
HVARY,NAME=SF1[K2],STEP=0.001,LOWER=-1.0,UPPER=0.0 ! Sextupole
HVARY,NAME=MF1[K3L],STEP=0.001,LOWER=0.0,UPPER=1.0 ! Multipole
```

The hFUnCTIONS command

```
HFUNCTIONS
```

computes the harmonic functions as defined in [12]. For the chromaticities and dispersions, but only for these, the effects of octupoles and decapoles are also considered and may be matched by varying those components. The HCELL command

```
HCELL, TOLERANCE=real,CALLS=integer,QX'=real,QY'=real,&
    QX','=real,QY''=real,QX','=real,QY','=real,&
    DQXDEX=real,DQYDEY=real,DQYDEX=real,&
    DX'I=real,DX''I=real,BX'I=real,BY'I=real,RXI=real,RYI=real,&
    DX 'S=real,DX ''S=real, BX 'S=real,BY'S=real,RXS=real, RYS=real
```

adjusts the following quantities to the values entered:
Tolerance Tolerance for the penalty function constructed.
CALLS Limit for the number of calls to the penalty function.
QX $\quad$ First-order horizontal chromaticity.
QY $\quad$ First-order vertical chromaticity.
QX, $\quad$ Second-order horizontal chromaticity.
QY, ${ }^{\prime} \quad$ Second-order vertical chromaticity.
QX, ', Third-order horizontal chromaticity.
QY ${ }^{\prime}, \quad$ Third-order vertical chromaticity.
DQXDEX Derivative of horizontal tune with horizontal emittance.
DQYDEY Derivative of vertical tune with vertical emittance.
DQYDEX Derivative of vertical tune with horizontal emittance.
DX'I Second-order horizontal dispersion at interaction point.
DX' 'I Third-order horizontal dispersion at interaction point.
BX'I Variation of $\beta_{x}$ with energy at interaction point.
BY' I Variation of $\beta_{y}$ with energy at interaction point.
RXI Contributions of horizontal resonances at interaction point.

Contributions of vertical resonances at interaction point.
DX'S Second-order horizontal dispersion at symmetry point.
DX''S Third-order horizontal dispersion at symmetry point.
BX'S Variation of $\beta_{x}$ with energy at symmetry point.
BY'S Variation of $\beta_{y}$ with energy at symmetry point.
RXS Contributions of horizontal resonances at symmetry point.
RYS Contributions of vertical resonances at symmetry point.
The chromaticities and dispersions of all orders calculated in HARMON contain the effects of multipoles up to the decapole. These multipole strengths may all be adjusted to minimise the distorsions. All quantities are fitted in a least-squares sense. The sextupole components to be varied must be specified in a HVARY command. The default matching weights are one for the first-order chromaticities, and zero for all other quantities. Different weights can be set before calling HCELL with the HWEIGHT command

```
HWEIGHT,QX'=real,QY'=real,QX''=real,QY''=real,QX',',=real,QY','=real,&
    DQXDEX=real,DQYDEY=real,DQYDEX=real,&
    DX'I=real,DX''I=real,BX'I=real,BY'I=real,RXI=real,RYI=real,&
    DX'S=real,DX''S=real,BX'S=real,BY'S=real, RXS=real,RYS=real
```

 RXI RYI RXS RYS

### 11.5 Example for a HARMON Sequence

```
USE, OCT, SUPER=4,SYMM
BEAM, PARTICLE=ELECTRON,ENERGY=50,&
    EX=0.0645E-6,EY=0.03225E-6,SIGE=1.25E-3
!
HARMON, FX=10,FY=10,FE=7
!
HCHROMATICITY
HVARY, NAME=SF1,STEP=0.001 ! Chromaticity sextupoles
HVARY, NAME=SD1,STEP=0.001
HTUNE, QX'=0,QY'=0
!
HRESONANCE,ORDER=3
!
HWEIGHT,QX ''=1.0,QY''=1.0,QX',''=10.0,QY',''=10.0
HVARY, SF2,STEP=0.001,LOWER=0.,UPPER=1.
HVARY, SD2,STEP=0.001,LOWER=-1.,UPPER=0.
HVARY, SF3,STEP=0.001,LOWER=0.,UPPER=1.
HVARY, SD3,STEP=0.001,LOWER=-1.,UPPER=0.
HCELL ! Chromaticities minimised to all three orders
!
HRESONANCE
ENDHARM
```

Table 12.1: Matching Commands

| Name | Function | Section |
| :--- | :--- | :--- |
| CELL | Initialise cell matching | 12.1 |
| MATCH | Initialise insertion matching | 12.1 |
| WEIGHT | Set matching weights | 12.2 |
| CONSTRAINT | Impose matching constraint | 12.3 |
| COUPLE | Impose periodicity between two points | 12.3 |
| VARY | Vary parameter | 12.2 |
| FIX | Fix parameter value | 12.2 |
| RMATRIX | Constraint on linear matrix | 12.3 |
| TMATRIX | Constraint on second-order terms | 12.3 |
| LEVEL | Set print level | 12.4 |
| LMDIF | Minimisation by gradient method | 12.5 |
| MIGRAD | Minimisation by gradient method | 12.5 |
| SIMPLEX | Minimisation by simplex method | 12.5 |
| ENDMATCH | Leave matching mode | 12.1 |

Before a match operation a beam line must be selected by means of a USE command (see Section 5.1). Matching of this line is then initiated by a CELL or MATCH command. From either of these commands to the corresponding ENDMATCH command MAD recognises the matching commands listed in Table 12.1. For a mathematical description of the minimisation procedures see [19]. In particular one may do the following:

- Define parameter(s) to be varied,
- Couple and/or set parameter values,
- Define constraints,
- Select desired printout detail,
- Match by different methods.

The matching commands are described in detail below. Some other commands can also be issued during matching.

### 12.1 Activation and Deactivation of the Matching Module

Before matching a beam line must be selected by means of a USE command (see Section 5.1). The type of match desired is then defined by entering a CELL or MATCH command.

### 12.1.1 Matching a Periodic Cell

In the first mode, called cell matching, a periodic cell is adjusted. The periodicity is enforced exactly, and constraints are fulfilled in the least squares sense. Cell matching mode is initiated by the CELL command:

```
CELL,DELTAP=real,ORBIT
```

It has two attributes:
DELTAP The value of the momentum error $\Delta p / p_{0} c$ for which the match should be performed (default: 0 ).

ORBIT If this flag is true, the closed orbit is also matched.
Examples:

```
! Match a simple periodic cell
USE,PERIOD=0CTANT,RANGE=CELL1
CELL,ORBIT !
! Match a symmetric and periodic cell with repetitions
USE , HALFCELL , SYMM , SUPER=5
CELL
```


### 12.1.2 Insertion Matching

In the second mode, called insertion matching, a beam line is matched with given initial values for the optical functions. Constraints may be imposed in other places, i.e. intermediate or end values can be requested. In this case the initial values are assumed as exact, and constraints are fulfilled in the least squares sense. The insertion matching mode is initiated by the MATCH command. In the simplest form the initial values for the optical functions are taken from the periodic solution of another beam line. In this case, all of ( $\beta_{x}, \beta_{y}, \alpha_{x}, \alpha_{y}, D_{x}, D_{y}, D_{p_{x}}, D_{p_{y}}$ ) are transmitted to be used as initial values. If there is a constraint on the orbit, the values $\left(x, p_{x}, y, p_{y}\right)$ of the orbit are also transmitted. Such a condition is entered in the form

```
MATCH, LINE=beam-line,MUX=real, MUY=real, DELTAP=real, ORBIT
```

The initial phases angles may be specified by:
MUX The initial horizontal phase $\mu_{x}$,
MUY The initial vertical phase $\mu_{y}$.
DELTAP The value of the momentum error $\Delta E / p_{0} c$ for which the match should be performed (default: 0 ).

ORBIT If this flag is true, the closed orbit is also matched.
Example:

```
CELL1: LINE=(...)
INSERT: LINE=(...)
        USE,INSERT
        MATCH,LINE=CELL1,MUX=9.345,MUY=9.876,0RBIT
```

This matches the beam line INSERT. Initial conditions are given by the periodic solution for the beam line CELL1.

It is also possible to enter numerical initial values. The MATCH command then has the form

```
MATCH, BETX=real,ALFX=real,MUX=real,&
    BETY=real,ALFY=real,MUY=real,&
    X=real,PX=real,Y=real,PY=real,&
    DX=real,DY=real,DPX=real,DPY=real,&
    DELTAP=real,ORBIT
```

It accepts as attributes the linear lattice functions listed in Section 1.5.3) and the orbit coordinates listed in Section 1.5.1. Omitted initial values are assumed to be zero. This implies that at least BETX and BETY are required to obtain physically meaningful results. Example:

```
USE, INSERT
MATCH, BETX=1.6,BETY=0.1
```

This matches the beam line INSERT with given initial values. Values not specified are set to zero.
The end conditions of a previous beam line can finally be transmitted via a SAVEBETA command (see also section 5.6.3):

```
USE,LINE1
SAVEBETA,LABEL=XYZ,PLACE=#E
TWISS
USE,LINE2
MATCH,BETAO=XYZ
```

This example transmits all values (including phases) as initial values. The orbit is used only if the flag ORBIT is entered. Some values may also be changed explicitly:

```
USE,LINE1
SAVEBETA,LABEL=XYZ,PLACE=#E
TWISS
USE,LINE2
MATCH , BETAO=XYZ , MUX=1.234, MUY=4.321
```

will overwrite the phases taken from the previous line.

### 12.1.3 End of Matching Run

The ENDMATCH command terminates the matching section and deletes all tables related to a tracking run:

```
ENDMATCH
```


### 12.2 Variable Parameters

A parameter to be varied is specified by the VARY command:

> VARY, NAME=variable,STEP=real,LOWER=real,UPPER=real

It has four attributes:
NAME $\quad$ The name of the parameter or attribute to be varied (see Section 2.4.10),
STEP The approximate initial step size for varying the parameter. If the step is not entered, MAD tries to find a reasonable step, but this may not always work.

LOWER Lower limit for the parameter (optional),
UPPER Upper limit for the parameter (optional).
Examples:

```
VARY,PAR1,STEP=1.0E-4 ! vary global parameter PAR1
VARY,QL11[K1],STEP=1.OE-6vary attribute K1 of the QL11
VARY,Q15[K1],STEP=0.0001,LOWER=0.0,UPPER=0.08 ! vary with limits
```

If the upper limit is smaller than the lower limit, the two limits are interchanged. If the current value is outside the range defined by the limits, it is brought back to range. After a matching operation all varied attributes retain their last value. They are never reset to an old value. Parameters cannot be varied if they depend on other parameters. Example:

```
P1:=10.0-P2
```

Here P2 may be varied, but P1 may not. The reason is that MAD is not able to compute P2 in terms of P1.

The command FIX removes a parameter or attribute from the table of variable parameters:

```
FIX,NAME=variable
```


## Example:

```
FIX ,NAME=QF[K1]
```


### 12.3 Constraints

### 12.3.1 Simple Constraints

Simple constraints are imposed by the CONSTRAINT command. It can take three forms. The first form is

```
CONSTRAINT, PLACE=range, LINE=beam-line, MUX=real, MUY=real
```

Here all of ( $\beta_{x}, \beta_{y}, \alpha_{x}, \alpha_{y}, D_{x}, D_{y}, D_{p_{x}}, D_{p_{y}}$ ) are constrained in range to the corresponding values of beam-line. If any of the six components of the closed orbit of the beam line is constrained, the command implies setting of the ORBIT flag. The phase advances ( $\mu_{x}, \mu_{y}$ ) may also be specified on the constraint. If the match succeeds, the part of the beam line from its beginning to place has the specified phase advance. The optical functions are such that beam-line could be inserted in place without a mismatch. If beam-line was defined with formal arguments, an actual argument list must be provided. Normally place refers to a single observation point (see Section 4.7).

The second form of the CONSTRAINT command is
CONSTRAINT, PLACE=range, BETAO=beta0-name, MUX=real , MUY=real

Here all of ( $\beta_{x}, \beta_{y}, \alpha_{x}, \alpha_{y}, D_{x}, D_{y}, D_{p_{x}}, D_{p_{y}}$ ) are constrained in range to the corresponding values of a pre-calculated BETAO module (see 5.6.3). If the ORBIT flag is set, then the six components of the closed orbit of the beam line are also constrained. Note that the default weights for T or PT are zero. The phases $\left(\mu_{x}, \mu_{y}\right)$ are not taken from the BETAO block. If they are to be constrained, they must be explicitly specified on the CONSTRAINT command. However, it is permitted to specify the phase values from the BETAO block:
CONSTRAINT , PLACE= . . . , BETAO=DATA , MUX=DATA [MUX] , MUY=DATA [MUY]

If the match succeeds, the part of the beam line from its beginning to place has the specified phase advance. Normally place refers to a single observation point (see Section 4.7).

The third form

```
CONSTRAINT, PLACE=place,&
    BETX=real,ALFX=real,MUX=real,&
        BETY=real,ALFY=real,MUY=real,&
        X=real,PX=real, Y=real, PY=real,T=real,PT=real,&
        DX=real,DPX=real, DY=real, DPX=real
```

allows one to enter numerical values for any optical function(s) in place. Place may refer to a range, denoted by two positions or by a beam line name. The constraint then applies to all positions within the range. The linear lattice functions listed in Section 1.5 .3 can all be constrained. The matching weights can be set by the WEIGHT command (see below). Upper and lower limits can be requested by replacing the equals sign by a less than sign $(<)$ or by a greater than sign ( $>$ ). Both lower and upper limits may be requested in the same command. If only lower and upper limits are used in a constraint, the corresponding place may be a range like e.g. a beam line. Example:

```
INSERT: LINE=(...)
CELL1: LINE=(...)
    USE, INSERT
    MATCH,LINE=CELL1
    CONSTRAINT, INSERT,DX<2.5,DX>0.0
```

Note that

### 12.3.2 Sub-Period Constraint

It is sometimes desired to make the linear lattice parameters equal in two places without imposing their actual values. This is requested by the COUPLE command

```
COUPLE, RANGE=range, MUX=real, MUY=real
```

where range refers to an observation range (see Section 4.7). MAD tries to adjust the matched beam line such that the functions $\left(\beta_{x}, \beta_{y}, \alpha_{x}, \alpha_{y}, D_{x}, D_{y}, D_{p_{x}}, D_{p_{y}}\right.$ ) are equal in the two end points of the range. The orbit cannot be constrained by this command. Optionally the phase advances over the range may be specified by means of the attributes MUX and/or MUY. The matching weights are taken from the latest WEIGHT command (see below). Example:

```
M: MARKER
INSERT: LINE=(...,M,...,M,...)
    USE, INSERT
    MATCH, BETX=2, BETY=1
    COUPLE ,M[1]/M[2], MUX=0.5
```


### 12.3.3 Constraints on Transfer map

Often the constraints are desired on the transfer map rather than on the lattice functions. The two commands

```
RMATRIX,RANGE=range {,RM(i,k)=real} {,WEIGHT(i,k)=real}
TMATRIX,RANGE=range {,TM(i,k,l)=real} {,WEIGHT(i,k,l)=real}
```

are provided for this purpose. They have the following attributes:
RANGE Denotes the beginning and end of the range over which the transfer map is to be fitted.
RM(i,k) A real array of dimension ( 6,6 ). As many elements of the linear transfer matrix can be constrained.
$\operatorname{TM}(i, k, 1) \quad$ A real array of dimension $(6,6,6)$. As many elements of the linear transfer matrix can be constrained.

WEIGHT A real array of dimension $(6,6)$ or $(6,6,6)$ respectively. If given, this specifies the matching weights for the corresponding matrix elements (default: 1 ).

### 12.3.4 Matching Weights

The matching procedures try to fulfil the constraints in a least square sense. A penalty function is constructed which is the sum of the squares of all errors, each multiplied by the specified weight. The larger the weight, the more important a constraint becomes. The weights are taken from a table of current values. These are initially set to default values, and may be reset at any time to different values. Values set in this way remain valid until changed again. The WEIGHT command

```
WEIGHT, BETX=real,ALFX=real,MUX=real,&
    BETY=real,ALFY=real,MUY=real,&
    X=real,PX=real,Y=real,PY=real,T=real,PT=real,&
    DX=real,DPX=real, DY=real, DPY=real
```

changes the weights for subsequent constraints. The weights are entered with the same name as the linear lattice function to which the weight applies, as listed in Section 1.5. The default weights are listed in Table 12.2. Note that matching T or PT always requires a WEIGHT command, since these quantities have zero default weights. Frequently the matching weights serve to select a restricted set of functions to be matched in a LINE=line constraint. One may match the dispersion at a given place to the dispersion of a line as in the following example:

```
WEIGHT, BETX=0.0, BETX=0.0, ALFX=0.0, ALFY=0.0,MUX=0.0,MUY=0.0
CONSTRAINT, #E,LINE=CELL
```

Table 12.2: Default Matching Weights

| name | weight | name | weight | name | weight | name | weight |
| :--- | :---: | :--- | :---: | :--- | :---: | :--- | :---: |
| BETX | 1.0 | ALFX | 10.0 | MUX | 10.0 |  |  |
| BETY | 1.0 | ALFY | 10.0 | MUY | 10.0 |  |  |
| X | 10.0 | PX | 100.0 | Y | 10.0 | PY | 100.0 |
| T | 0.0 | PT | 0.0 |  |  |  |  |
| DX | 10.0 | DPX | 100.0 | DY | 10.0 | DPY | 100.0 |

### 12.4 Matching Output Level

The LEVEL command sets the output level for matching:

```
LEVEL, LEVEL=integer
```

Recognised level numbers are:
0 Minimum printout: Messages and final values only,
1 Normal printout: Messages, initial and final values,
2 Like LEVEL=1, plus every tenth new minimum found,
3 Like LEVEL=2, plus every new minimum found,
4
Like LEVEL=3, plus eigenvalues of covariance matrix (MIGRAD method only).
Example:
LEVEL, 2

### 12.5 Matching Methods

### 12.5.1 LMDIF, Gradient Minimisation

The LMDIF command minimises the sum of squares of the constraint functions using their numerical derivatives:

```
LMDIF,CALLS=integer,TOLERANCE=real
```

It is the fastest minimisation method available in MAD. The command has two attributes:
CALLS The maximum number of calls to the penalty function (default: 1000).
TOLERANCE The desired tolerance for the minimum (default: $10^{-6}$ ).
Example:
LMDIF, CALLS=2000,TOLERANCE=1.OE-8

### 12.5.2 MIGRAD, Gradient Minimisation

The MIGRAD command minimises the penalty function using its numerical derivatives of the sum of squares:

MIGRAD, CALLS=integer, TOLERANCE=real, STRATEGY=1
The command has two attributes:
CALLS The maximum number of calls to the penalty function (default: 1000).
TOLERANCE The desired tolerance for the minimum (default: $10^{-6}$ ).
STRATEGY A code for the strategy to be used (default: 1). The user is referred to the MINUIT manual for explanations [19].

Example:
MIGRAD, CALLS $=2000$, TOLERANCE $=1.0 E-8$

### 12.5.3 SIMPLEX, Minimisation by Simplex Method

The SIMPLEX command minimises the penalty function by the simplex method:

```
SIMPLEX,CALLS=integer,TOLERANCE=real
```

Details are given in the description of the MINUIT program [19]. The command has two attributes:
CALLS The maximum number of calls to the penalty function (default: 1000).
TOLERANCE The desired tolerance for the minimum (default: $10^{-6}$ ).
Example:
SIMPLEX, CALLS=2000,TOLERANCE=1.OE-8

### 12.6 Matching Examples

### 12.6.1 Simple Periodic Beam Line

Match a simple cell with given phase advances:

```
QF: QUADRUPOLE,...
QD: QUADRUPOLE,...
CELL1: LINE=(...,QF,...,QD,...)
    USE,CELL1
    CELL
    VARY ,NAME=QD [K1],STEP=0.01
    VARY,NAME=QF[K1],STEP=0.01
    CONSTRAINT,PLACE=#E,MUX=0.25,MUY=1/6
    MIGRAD,CALLS=2000
    ENDMATCH
```


### 12.6.2 Insertion Matching

Match an insertion INSERT to go between two different cells CELL1 and CELL2:

```
CELL1: LINE=(...)
CELL2: LINE=(...)
INSERT: LINE=(...)
    USE,INSERT
    MATCH,LINE=CELL1
    VARY,...
    CONSTRAINT , PLACE=#E , LINE=CELL2 , MUX=. . , MUY=. . .
    SIMPLEX
    MIGRAD
    ENDMATCH
```

Adjust the beam line INSERT such that it has specified values of the optical functions at its beginning and matches the line CELL7 at its end:

```
CELL7: LINE=(...)
INSERT: LINE=(...)
    USE, INSERT
    MATCH,BETX=1.6,BETY=0.1
    VARY,...
    CONSTRAINT, PLACE=#E,LINE=CELL7
    SIMPLEX
    MIGRAD
    ENDMATCH
```


### 12.6.3 A Matching Example for LEP, Version 11

Here the end values are required to be exact. This may be achieved by inverting the beam line:

```
! Define beam lines
CELL: LINE=(QDH,L23,SD,L22,B6,L21,QF,L23,SF,L22,B6,L24,QDH)
MARK: MARKER
DISS: LINE=(QS11,L25,BW,L24,QS12,L25,B4,L24,QS13,L25,B4,&
    L24,QS14,L25,B4,L31,QS15,L25,B4,L32,SF2,L23,QS16)
```

```
RFS: LINE=(L5,QS5,L5,QS6,L5,2*(QS7H,MARK,QS7H,L5,QS8,L5))
LOBS: LINE=(L1,QS1,L2,QS2,L3,QS3,L4,QS4)
INSS: LINE=(LOBS,RFS,DISS,L21,B6,L22,QDH)
        USE, (-INSS)
!
! Call matching module and define initial values
MATCH,LINE=CELL
!
! Variable parameters
    VARY,QS1[K1],STEP=0.01,LOWER=-0.06,UPPER=0.06
    VARY,QS2[K1],STEP=0.01,LOWER=-0.06,UPPER=0.06
    VARY,QS3[K1],STEP=0.01,LOWER=-0.06,UPPER=0.06
    VARY,QS4[K1],STEP=0.001,LOWER=-0.06,UPPER=0.06
    VARY,QS5[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS6[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS7[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS8[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS11[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS12[K1],STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS13[K1], STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY, QS14[K1] ,STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY,QS15[K1], STEP=0.01,LOWER=-0.035,UPPER=0.035
    VARY, QS16[K1], STEP=0.01,LOWER=-0.035,UPPER=0.035
!
! Constraints at the interaction point
    WEIGHT, BETX=10.0, BETY=50.0, ALFX=10.0, ALFY=10.0,
    MUX=10.0,MUY=10.0,DX=10.0,DPX=100.0
    CONSTRAINT, #E, BETX=1.6, BETY=0.1, ALFX=0.0, ALFY=0.0,
    MUX=1.771875,MUY=2.0125,DX=0.0,DPX=0.0
!
! Constraints at two intermediate points (MARK occurs twice)
    CONSTRAINT, MARK, ALFX=0.0, ALFY=0.0,DX=0.0,DPX=0.0
    COUPLE,MARK [1]/MARK [2]
!
! Constraint for limits of dispersion in dispersion suppressor
    CONSTRAINT,DISS,DX>0,DX<1.25
!
! Perform match
    SIMPLEX,CALLS=1000
    MIGRAD, CALLS =24000, TOLERANCE=1.OE-9
!
! End of matching procedure
ENDMATCH
```


### 12.6.4 Examples for Complex Matching Constraints

All initial conditions and constraint values can depend on variable parameters. This feature permits to set up very complex matching conditions. The first example shows how an insertion can be designed which exchanges the values of the $\beta$-functions for the two transverse planes in two places M1 and M2:

```
M1: MARKER
M2: MARKER
INSERT: LINE=(...,M1,...,M2,...)
    CONSTRAINT,M1,BETX=BETA1,ALFX=ALFA1,BETY=BETA2,ALFY=ALFA2
    CONSTRAINT, M2, BETX=BETA2,ALFX=ALFA2, BETY=BETA1,ALFY=ALFA1
    VARY,BETA1,STEP=0.01
    VARY,BETA2,STEP=0.01
    VARY,ALFA1,STEP=0.01
    VARY,ALFA2,STEP=0.01
```

The trick is to match the lattice functions in M1 to the four variable parameters

```
BETA1,ALFA1,BETA2,ALFA2
```

and in M2 to the same parameters in a different order.
The second example is taken from a spin matching problem.

```
! Match towards interaction point
MULT:=(1+SINTHETA)/SINTHETA ! A constant multiplication factor.
MATCH,LINE=SRF3M
! Lattice functions imposed at interaction point
WEIGHT, BETX=1, BETY=10,DY=30,DPY=300
CONSTRAINT, LORIPS, BETX=1.75,BETY=0.07,ALFX=0, ALFY=0,DY=0
! ***** Spin matching constraints *****
RMATRIX,DBB[1]/LORIPS, RM (3,4)=0, RM (2, 1)=AUX
VARY,AUX,STEP=0.01
RMATRIX,B2UP[1]/LORIPS,RM(3,4)=0,RM (2, 1)=MULT*AUX
! Quadrupoles to be varied
VARY,KQSOMP,STEP=0.01,LOWER=-0.2,UPPER=0.2
VARY,KQS1MP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS3MP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS4MP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS5AP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS6AP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS5BP,STEP=0.01,LOWER=-0.1,UPPER=0.1
VARY,KQS6BP,STEP=0.01,LOWER=-0.1,UPPER=0.1
! Initiate matching
SIMPLEX,CALLS=1000,TOLE=1E-3
LMDIF, CALLS=6000,TOLE=1E-12
ENDMATCH
```

Here the interesting point lies in the spin matching constraints. The conditions RM $(3,4)=0$ on the two RMATRIX commands ensure the proper phase advances for the vertical phase ( 0.5 and 1.5 respectively). The conditions on $\operatorname{RM}(2,1)$ enforce the condition

$$
A_{21}=\frac{(1+\sin \theta)}{\sin \theta} C_{21}
$$

where $A$ and $C$ are the transfer matrices from the centre of either vertical bend to the interaction point. Since a direct coupling between the values of the transfer matrix in two different points is not
possible, a freely variable parameter AUX is introduced. The value of $C_{21}$ is matched to AUX, while the value of $A_{21}$ is matched to an expression in AUX.

# Chapter 13. Tracking Module 

Table 13.1: Tracking Commands

| Name | Function | Section |
| :--- | :--- | :--- |
| NOISE | Define noise on parameters | 13.2 |
| OBSERVE | Define observation point | 13.3 |
| START | Define initial coordinates | 13.4 |
| RUN | Start tracking | 13.6 |
| TSAVE | Save particle positions | 13.5 |
| ENDTRACK | Terminate tracking mode | 13.1 |

Before starting to track, the working beam line must be selected by means of a USE command (see Section 5.1). The energy and emittances should be defined by a BEAM command (see Section 5.2). Tracking is then initiated by the TRACK command. From this command to the corresponding ENDTRACK command MAD accepts the tracking statements listed in Table 13.1. Tracking is done in parallel i.e. the coordinates of all particles are transformed at each beam element as it is reached.

### 13.1 Activation and Deactivation of the Tracking Module

Tracking through a beam line is initiated by the TRACK command. It has the form

```
TRACK [,ONEPASS] [,DAMP] [,QUANTUM]
```

The emittances and $\sigma$ 's are evaluated from the latest BEAM command. A BEAM command should not occur while the tracking module is active. The TRACK command has three attributes:

ONEPASS This flag tells MAD not to compute the normalisation transformations and to assume that the machine is stable.

The two other flags have only effect if synchrotron radiation is switched on on the BEAM command:
DAMP This flag causes MAD to consider the systematic component (bias) of energy loss by synchrotron radiation.

QUANTUM If this flag is present together with DAMP, it causes MAD to simulate the quantum effects of synchrotron radiation.

The former attributes RFCAVITY and DELTAP are ignored in this version of MAD. The TRACK command must be preceded by a EMIT command, which allows to adjust the average momentum error. The RF frequencies are set by TRACK according to this momentum error set in EMIT.

The synchrotron tune $Q_{s}$ computed must be non-zero, if $E_{t}$ or $\sigma_{e}$ is used on the BEAM command. Example for a TRACK command:

```
BEAM, PARTICLE=ELECTRON,ENERGY=50,RADIATE,&
    EX=0.0645E-6,EY=0.03225E-6,SIGE=1.25E-3
EMIT,DELTAP=0.01 TRACK,DAMP,QUANTUM
```

The ENDTRACK command terminates the tracking section:

```
ENDTRACK
```

It deletes all information relevant to the current tracking run.

### 13.2 NOISE Statement

One may define noise to be applied to magnet excitations with the NOISE statement

```
NOISE,VARIABLE=variable,AMPLITUDE(i)=real, FREQUENCY=real,PHASE=real
```

The NOISE statement must be entered after the TRACK, but before the RUN command. It has the effect to apply several sinusoidal variations to the specified parameter. The command has four attributes.

VARIABLE The variable to be affected.
AMPLITUDE (i) The value of the $i^{t h}$ noise amplitude $A_{i}$.
FREQUENCY (i) The value of the $i^{t h}$ noise frequency $f_{i}$.
PHASE (i) The value of the $i^{t h}$ noise phase $\phi_{i}$.
Before each turn is tracked, the noise is re-evaluated as

$$
\Delta A=\sum_{i=1}^{N} A_{i} \cos \left(2 \pi\left(f_{i} t+\phi_{i}\right)\right)
$$

where $t$ is the real time.

### 13.3 OBSERVE Statement

The statement OBSERVE

```
OBSERVE,PLACE=place,TABLE=table
```

defines a new observation point where a track table is to be generated. It has the attributes:
PLACE The observation point. Only the first occurrence in the machine is used.
TABLE The name to be given to the table.
The OBSERVE statement must be entered after the TRACK, but before the RUN command.

### 13.4 START Statement

The START command defines the initial coordinates of the particles to be tracked. There may be many START statements, one for each particle. Particles are always started with coordinates relative to the computed closed orbit for the defined energy error. The command format is:

```
START, X=real,PX=real,Y=real,PY=real,T=real,DELTAP=real,&
    FX=real,PHIX=real,FY=real,PHIY=real,FT=real,PHIT=real
```

The START statement must be entered after the TRACK, but before the RUN command.
For each canonical pair two specifications are possible. First, they may be specified as displacements with respect to the closed orbit: X, PX, Y, PY, T, DELTAP, as defined in Section 1.5.1. In this case the initial conditions are:

$$
\begin{aligned}
& x=\mathrm{X}, \quad p_{x} \quad=\mathrm{PX}, \\
& y=\mathrm{Y}, p_{y} \quad=\mathrm{PY}, \\
& \Delta t=\mathrm{T}, \quad \Delta E / c p_{0}=\text { DELTAP. }
\end{aligned}
$$

It is also possible to enter rotations in normalised phase space. Initial conditions $Z$ in normalised phase space are related to the closed orbit and normalised via the eigenvectors:

$$
\begin{aligned}
Z=Z_{c o} & +\sqrt{E_{x}} \mathrm{FX}\left(\Re V_{k} \cos \mathrm{PHIX}+\Im V_{k} \sin \mathrm{PHIX}\right) \\
& +\sqrt{E_{y}} \mathrm{FY}\left(\Re V_{k} \cos \mathrm{PHIY}+\Im V_{k} \sin \mathrm{PHIY}\right) \\
& +\sqrt{E_{t} \mathrm{FT}}\left(\Re V_{k} \cos \mathrm{PHIT}+\Im V_{k} \sin \mathrm{PHIT}\right)
\end{aligned}
$$

where $Z_{c o}$ is the closed orbit vector, $\Re V_{k}$ and $\Im V_{l}$ are the real and imaginary parts of the $k^{\text {th }}$ eigenvectors. The other variables have the meaning:

FX The normalised (betatron) amplitude for mode 1,
PHIX The (betatron) phase PHIX $=\phi_{x} / 2 \pi$ for mode 1 ,
FY The normalised (betatron) amplitude for mode 2,
PHIY $\quad$ The (betatron) phase PHIY $=\phi_{y} / 2 \pi$ for mode 2,
FT The normalised (synchrotron) amplitude for mode 3,
PHIT The (synchrotron) phase PHIT $=\phi_{t} / 2 \pi$ ) for mode 3.
The eigenvectors are computed in the TRACK command. The emittances are calculated in the latest EMIT command preceding it, or taken from the latest BEAM command, whichever comes last.

### 13.5 TSAVE Statement

The TSAVE command
TSAVE, FILENAME=string
saves the latest particle positions on the file named by string. These will normally be the positions of surviving particles after the last RUN command. If no RUN command has been seen yet, the positions are taken from any START commands seen. The positions are written as START commands, which may be read by a subsequent tracking run.

### 13.6 RUN Statement

This command starts or continues the actual tracking:

```
RUN,METHOD=TRANSPORT|LIE3|LIE4 [,TABLE=name]&
    TURNS=integer, FPRINT=integer, FFILE=integer
```

The RUN command initialises tracking and uses the values in the last particle bank for initial conditions. It optionally builds a table of particle positions, suited for plotting or further analysis; it may also write a portable binary file. Note that the CONTINUE command has been deleted from Version 8.2. RUN has the attributes:

METHOD The name of the tracking method to be used. It may be one of:
TRANSPORT The TRANSPORT method [6], using transfer matrices of order two (default),
LIE3 The Lie-algebra method [13] to order 2 in the canonical variables,

TABLE The name to be given to the tracking table. If a table with the same name already exists, it is discarded. If TABLE is omitted, no table is built. The table is not saved automatically; it can be saved on a file by the ARCHIVE command.

TURNS The number of turns (integer) to be tracked (default: 1).
FPRINT The print frequency, an integer (default: no printing). Printing always occurs before the first and after the last turn. If the value of FPRINT is greater than zero, printing also occurs after every FPRINT turns. This print-out includes normalised values, invariants, and phases (for their definitions see 13.4). Printing in other positions may be requested by a SELECT, TRACK command.

FFILE The output frequency for the binary file, an integer (default: no printing). If the value of FFILE is greater than zero, output occurs after every FFILE turns. The binary file is written in EPIO format, and has not been frozen yet. The output will be used by an off-line analysis program.

Example:

```
RUN,TURNS=1000,FPRINT=10
```

This RUN command tracks 1000 turns and prints every $10^{\text {th }}$ turn.

### 13.7 A Tracking Example

```
L: LINE=(OCT,-0CT)
USE, L
! Misalignments and closed orbit correction may be done here.
BEAM,EX=0.1E-6,EY=0.05E-6,ET=1.0E-3, ENERGY=60.0
TRACK
    START,X=0.001,Y=0.001,DELTAP=0.001 ! absolute X and Y
    START,FX=10.0,FY=10.0,DELTAP=-0.001! particle at 10 sigma
    RUN,TURNS=1024,FPRINT=1
    PLOT,HAXIS=X,VAXIS=PX,MULTIPLE
ENDTRACK
```

Plotting can be requested as described in section 1.5.6. The position of the PLOT command is irrelevant, as long as it occurs after the RUN command, and before any command which overwrites the track table generated.

# Chapter 14. Subroutines and Procedures 

Table 14.1: Subroutine Commands

| Name | Meaning | Section |
| :--- | :--- | :--- |
| SUBROUTINE | Begin subroutine definition | 14.1 |
| ENDSUBROUTINE | End subroutine definition | 14.1 |
| CALLSUBROUTINE | Call predefined subroutine | 14.2 |
| DO | Repeated execution of commands | 14.3 |
| ENDDO | End of DO group | 14.3 |

The input language provides features for gathering strings of statements, which may be treated as a simple command called a subroutine. There are commands to define a subroutine in terms of other commands, and to execute subroutines.

These language features allow the user to assemble a personal library of commonly used commands. Individual commands and subroutines may be called from this library whenever they are wanted, and simple labels may be substituted for relatively complicated subroutines or commands. The personal statement library can be appended to the MAD dictionary, which makes the statements available automatically when the program is started. Alternatively it can be placed in a user file and accessed on demand via the CALL and RETURN commands. Good candidates for a subroutine library are command sequences for the harmon, Matching, or Tracking Module.

### 14.1 SUBROUTINE and ENDSUBROUTINE

The SUBROUTINE and ENDSUBROUTINE statements delimit a group of related statements which together form the body of the subroutine. A subroutine definition, followed by a call, has the format

| label: | SUBROUTINE | ! begin of subroutine "label" |
| :--- | :--- | :--- |
|  | $\ldots$ | ! included statements |
|  | ENDSUBROUTINE | ! end of subroutine "label" |
|  | $\ldots$ | ! |
|  | other commands |  |
|  | label | call to subroutine "label" |

The subroutine is given the name label. The statements included are checked and stored in memory, but not executed until the subroutine is called. Example:

```
S: SUBROUTINE ! begin of definition for S
    USE,SSC
    SURVEY
    TWISS
ENDSUBROUTINE ! end of definition for S
S ! here S is executed the first time
```

The size of a subroutine is limited only by the available memory space. A labelled command within a subroutine may be called by its label from anywhere in the program. A subroutine cannot contain another SUBROUTINE, DO or ENDDO statement. However, within a subroutine, nested calls to other subroutines are permitted. Also, references to DO statements which have already been accepted by the program are permitted.

### 14.2 CALLSUBROUTINE

The CALLSUBROUTINE command causes execution of a group of commands previously assembled as a subroutine or procedure. It is included in the language for formal purposes only, to make the call explicit.

```
CALLSUBROUTINE,label
```

A statement consisting of label only is equivalent to CALLSUBROUTINE, label. The following are two equivalent calls to a subroutine $S$ :

```
S
CALLSUBROUTINE,S
```


### 14.3 DO and ENDDO

The DO statement allows grouping of one or more commands and offers an option to repeat their execution. Both subroutines and commands may be included. The list of commands follows the DO statement, and ends with an ENDDO:

```
DO,TIMES = integer]
    {command}
ENDDO
```

The commands SUBROUTINE, ENDSUBROUTINE, or DO must not be placed between DO and ENDDO. Example:

```
QUAD3: QUAD, L=1.5,K1=0.01
DO,TIMES=2
    TWISS,DELTAP=-0.01:+0.01:0.002
    SET, QUAD3[K1], QUAD3[K1]+0.001
ENDDO
```

This example shows how a SET statement may change parameters of the computations after each iteration. The length of a DO list is not limited.

## Chapter 15. Known Defects of Version 8

### 15.1 Definitions

MAD uses full $6 \times 6$ matrices to allow coupling effects to be treated, and the canonical variable set $x, p_{x} / p_{0}, y, p_{y} / p_{0},-c \Delta t, \Delta E / p_{0} c$, as opposed to other programs most of which use the set $x, x^{\prime}, y, y^{\prime}$, $-\Delta s, \delta$. Like the program MARYLIE [13], MAD uses the relative energy error $\Delta E / p_{0} c$ which is related to the relative momentum error $\delta$ by $\Delta E / p_{0} c=\beta \Delta p / p_{0}=\beta \delta$.

As from Version 8.13, MAD uses an additional constant momentum error $\delta_{s}=\Delta p / p_{0}$ in all optical calculations. The transfer maps contain the exact dependence upon this value; therefore the tunes for large deviations can be computed with high accuracy as opposed to previous versions.

The choice of canonical variables in MAD still leads to slightly different definitions of the lattice functions. In MAD the Courant-Snyder invariants [10] take the form $W_{x}=\gamma_{x} x^{2}-2 \alpha_{x} x p_{x}+\beta_{x} p_{x}^{2}$. Comparison to the original form $W_{x}=\gamma_{x} x^{2}-2 \alpha_{x} x x^{\prime}+\beta_{x} x^{\prime 2}$ shows that the orbit functions cannot be the same. A more detailed analysis, using $x^{\prime}=p_{x} /(1+\delta)$, shows that all formulas can be made consistent by defining the MAD orbit functions as

$$
\beta_{x M}=\beta_{x}(1+\delta), \quad \alpha_{x M}=\alpha_{x}, \quad \gamma_{x M}=\gamma_{x} /(1+\delta)
$$

For constant $\delta_{s}$ along the beam line and $\delta=0$, the lattice functions are the same. In a machine where $\delta$ varies along the circumference, e.g. in a linear accelerator or in an $\mathrm{e}^{+} \mathrm{e}^{-}$storage ring, the definition of the Courant-Snyder invariants must be generalised. The MAD invariants have the advantage that they remain invariants along the beam line even for variable $\delta$.

With the new method this problem occurs in TWISS only for non-constant $\delta$.

### 15.1.1 Treatment of $\partial^{2} Q / \partial \delta^{2}$ in TWISS

It has been noted in [23] that MAD returned tunes which are too low for non-zero $\Delta p$. The difference was found to be quadratic in $\Delta p$ with a negative coefficient. This problem has been eliminated thanks to the new treatment of momentum errors from Version 8.13 onwards.

## Appendix A. Format of the SURVEY and TWISS Files

All output generated by the TAPE options of the commands SURVEY or TWISS has the same general layout. The output is a coded file with default name SURVEY or TWISS. It consists of a header record, a set of position records, and a trailer record.

## A. 1 Common Output

In all cases the header record has two lines. It has the format
(5A8, I8, L8, I8/A80)
and contains:
PROGVRSN The current version number of MAD,
DATAVRSN This field may contain the following strings:
SURVEY Output of the SURVEY command follows,
TWISS Output of the TWISS command (linear lattice functions), follows
CHROM Output for the CHROM option of the TWISS command follows,
DATE The date of the MAD run. This is the same date as on the printed output.
TIME The time of the MAD run. This is the same time as on the printed output.
JOBNAME The job name for the MAD run, if available from the operating system.
SUPER The number of superperiods.
SYMM The symmetry flag. This is true, if only one half of a symmetric super-period was computed.

NPOS The number of position records that follow. This is larger by one than the number of elements.

TITLE The page header as defined by the latest TITLE command.
There are NPOS position records. Each of them contains four or five lines. The first two lines have the format
(A4, A16, F12.6, 3E16.9/5E16.9)
and contain the following information about the preceding element:
KEYWORD The element keyword,
NAME The element name,
Note that the TYPE parameter no longer occurs in this file. The remaining values are written according to Table A.1. The first position record refers to the position preceding the first element. Its content is
KEYWORD=blank, NAME='INITIAL'

All other values are zero. The remaining lines of the position records differ for the three cases.

Table A.1: Element Data in Position Records

| Keyword |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| DRIFT | L | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| RBEND | L | ANGLE | K1 | K2 | TILT | E1 | E2 | H1 | H2 |
| SBEND | L | ANGLE | K1 | K2 | TILT | E1 | E2 | H1 | H2 |
| QUADRUPOLE | L | 0 | K1 | 0 | TILT | 0 | 0 | 0 | 0 |
| SEXTUPOLE | L | 0 | 0 | K2 | TILT | 0 | 0 | 0 | 0 |
| OCTUPOLE | L | 0 | 0 | 0 | TILT | K3 | 0 | 0 | 0 |
| MULTIPOLE | 0 | KOL | K1L | K2L | T0 | K3L | T1 | T2 | T3 |
| SOLENOID | L | 0 | 0 | 0 | 0 | KS | 0 | 0 | 0 |
| RFCAVITY | L | 0 | 0 | 0 | 0 | FREQ | VOLT | LAG | 0 |
| ELSEPARATOR | L | 0 | 0 | 0 | TILT | EFIELD | 0 | 0 | 0 |
| KICKER | L | 0 | 0 | 0 | TILT | HKICK | 0 | 0 | 0 |
| HKICKER | L | 0 | 0 | 0 | TILT | KICK | 0 | 0 | 0 |
| VKICKER | L | 0 | 0 | 0 | TILT | KICK | 0 | 0 | 0 |
| SROT | 0 | 0 | 0 | 0 | 0 | ANGLE | 0 | 0 | 0 |
| YROT | 0 | 0 | 0 | 0 | 0 | ANGLE | 0 | 0 | 0 |
| MONITOR | L | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| HMONITOR | L | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| VMONITOR | L | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MARKER | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## A. 2 SURVEY Output

When DATAVRSN=SURVEY, the third and fourth lines of the position records have the format
(4E16.9/3E16.9)
and contain the global coordinates and the cumulative length in the order

| $X$ | $Y$ | $Z$ | SUML |
| :--- | :--- | :--- | :--- |
| THETA | PHI | PSI |  |

The trailer record has two lines in the format
(3E16.9/3E16.9)
and contains the coordinates of the machine centre, the minimum and maximum radius and the machine circumference in the order

| X | $Y$ | $Z$ |
| :--- | :--- | :--- |
| RMIN | RMAX | $C$ |

## A. 3 TWISS Output

When DATAVRSN=TWISS, the third to fifth lines of the position records have the format
(5E16.9/5E16.9/5E16.9)
and contain the quantities

| ALFX | BETX | MUX | DX | DPX |
| :--- | :--- | :--- | :--- | :--- |
| ALFY | BETY | MUY | DY | DPY |
| X | PX | $Y$ | PY | SUML |

in this order. The trailer record has three lines in the format
(3E16.9/5E16.9/5E16.9)
and contains

| DELTAP | GAMTR | C |  |  |
| :--- | :--- | :--- | :--- | :--- |
| COSMUX | QX | QX, | BXMAX | DXMAX |
| COSMUY | QY | QY, | BYMAX | DYMAX |

in this order. If the COUPLE option was given, the output has the same format, but the quantities given refer to the two possible modes $(1,2)$ instead of referring to the two planes $(x, y)$.

## A. 4 CHROM Output

When DATAVRSN=CHROM, the third to fifth lines of the position records have the format
(5E16.9/5E16.9/5E16.9)
and contain

| WX | PHIX | DMUX | DDX | DDPX |
| :--- | :--- | :--- | :--- | :--- |
| WY | PHIY | DMUY | DDY | DDPY |
| X | PX | $Y$ | PY | SUML |

in this order. There is no trailer record in this case.

## Appendix B. Format for File Names

The MAD program normally uses alphanumeric names for files. This Appendix describes the valid formats for file names, as used in different computer operating systems.

## B. 1 IBM VM/CMS System

## B.1.1 Standard Streams

The standard streams are defined within MAD with the names listed in Table B.1. At CERN, the

Table B.1: Standard Files Used by MAD, IBM-VM/CMS Version

| Purpose | unit | file name |
| :--- | :--- | :--- |
| Command dictionary input | 4 | DICT |
| Normal input | 5 |  |
| Input lines and error messages | 6 |  |
| Plot output (GKS metafile) | 8 | MAD METAFILE |
| Plot output (HIGZ metafile) | 8 | MAD PS |
| Normal output | 14 | PRINT |
| Dynamic tables | 15 | TABLE |

EXEC file MAD8 EXEC stored on the MAD disk performs the following default initialisations:

```
FILEDEF DICT DISK MAD8 DICT * FILEDEF 5 DISK TERMINAL
FILEDEF 6 DISK TERMINAL
FILEDEF PRINT DISK problem LISTING scratch
FILEDEF DUMP DISK problem DUMP scratch
FILEDEF METAFILE DISK MADPLOT METAFILE scratch
```

Problem is the name of the input file (or PROBLEM in interactive mode). Scratch is the mode of a scratch disk created by the EXEC file. The behaviour of MAD can be adapted to the user's needs by changing the EXEC file.

## B.1.2 User-Defined Files

All other file names must be chosen according to the VM/CMS conventions. Within MAD a file name has one to three parts separated by dots

```
filename.filetype.filemode
filename.filetype
    filetype
```

All letters are converted to uppercase. If all three parts are present, the file name is used as entered. Example:

```
CALL,FILENAME='LEPTWO.DATA.D'
```

If two parts are present, they are taken as filename and filetype. The filemode defaults to * for input files, or to the contents of the REXX variable SCRATCH for output files. SCRATCH The CERN EXEC MAD8 EXEC sets this variable to the mode of a scratch disk. Examples:

```
CALL, FILENAME='TEST.MAD'
TWISS, TAPE='LEP.TWISS'
```

Assume that the mode of the scratch disk is F. CALL requires an input file and TWISS an output file, so these commands have the same effect as:

```
CALL, FILENAME='TEST MAD *'
TWISS, TAPE='LEP TWISS F'
```

If only one part appears, it is used as filetype. The filemode has the same default as above, and filename defaults to the contents of the REXX variable PROBLEM. The CERN EXEC MAD8 EXEC sets this variable to the filename of the DATA file, or if MAD runs interactively, to the value PROBLEM. Example:

```
CALL, FILENAME='DATA'
CALL ! FILENAME='SAVE' is default
TWISS, TAPE ! FILENAME='TWISS' is default
TWISS, TAPE='LATTICE'
```

Assume that the filename of the DATA file is LEP and the mode of the scratch disk is F. The above examples have then the same effect as:

```
CALL, FILENAME='LEP DATA *'
CALL, FILENAME='LEP SAVE *'
TWISS, TAPE='LEP TWISS F'
TWISS, TAPE='LEP LATTICE F'
```


## B. 2 UNIX and UNICOS Systems

## B.2.1 Standard Stream Names

The standard streams are defined within MAD with the names listed in Table B.2. Note that on the Cray XMP the table file is not used; dynamic tables reside in SSD. File names are converted to

Table B.2: Standard Files Used by MAD, UNIX Version

| Purpose | unit | file name |
| :--- | :--- | :--- |
| Command dictionary input | 4 | dict |
| Normal input | 5 | stdin |
| Input lines and error messages | 6 | stdout |
| Plot output (GKS metafile) | 8 | mad.metafile |
| Plot output (HIGZ metafile) | 8 | mad.ps |
| Normal output | 14 | print |
| Dynamic tables | 15 | table |

upper case, unless they are enclosed in quotes. Without special action they refer to files in the user's working directory. They can be assigned other names by the ln command entered at the keyboard or issued in a shell script. Such a link is desired in particular for the command dictionary:

```
ln /usr/madman/mad.dict dict
```

This link makes the command dictionary available under the assumption that it resides in the file $/ \mathrm{usr} / \mathrm{madman} / \mathrm{mad} . \mathrm{dict}$. It is recommended to create a subdirectory for each problem, and to store the data file in this subdirectory. MAD should then be run while this directory is current, so as to create all output files in the same directory. This helps keeping related files together.

## B.2.2 User-Defined Files

Other files can be assigned any name accepted by the UNIX system, with the restriction that MAD accepts at most 40 characters, and that it converts file names to upper case unless enclosed in quotes. Examples:

```
CALL, FILENAME='ssc.data'
TWISS, TAPE='ssc/twiss.data'
```

The first file is read in the user's working directory, and the TWISS output is written on the file twiss.data in the subdirectory ssc.

## B. 3 VAX VMS System

## B.3.1 Standard Stream Names

The standard streams are defined within MAD with the names listed in Table B.3. Without special
Table B.3: Standard Files Used by MAD, VAX-VMS Version

| Purpose | unit | file name |
| :--- | :--- | :--- |
| Command dictionary input | 4 | DICT |
| Normal input | 5 |  |
| Input lines and error messages | 6 |  |
| Plot output (GKS metafile) | 8 | METAFILE |
| Plot output (HIGZ metafile) | 8 | MAD.PS |
| Normal output | 14 | PRINT |
| Dynamic tables | 15 | TABLE |

action, these file names refer to files in the user's working file directory. They can be assigned other names by an ASSIGN command entered at the keyboard or issued in a DCL file. This is desirable in particular for the command dictionary:

ASSIGN DISK\$SI:[PUBSI]MAD8.DICT DICT
This command makes the dictionary available on the CERN central VAX system (VXCERN). The other standard streams should be assigned by a sequence like

```
ASSIGN DISK$gg:[user]mad.print PRINT
ASSIGN DISK$gg:[user]mad.metafile METAFILE
```


## B.3.2 User-Defined Files

Other files can be assigned any name accepted by the VMS system, with the restriction that MAD accepts at most 40 characters. All letters are converted to upper case. Examples:

```
CALL, FILENAME='SSC.DATA'
TWISS, TAPE='DISK$IZ[MADMAN]SSC.TWISS'
SURVEY, TAPE='DISK$IZ[MADMAN.SSC]SURVEY.DATA'
```

The first file is read in the user's working directory; the TWISS output is written on a file SSC.TWISS in the current directory, and the SURVEY output is written on a file SURVEY. DATA in the subdirectory SSC.

## Appendix C. Format of TFS Files

TFS files (Table File System) are used in the LEP control system. Their use is documented in Reference [11]. The MAD program knows only coded TFS files. This Appendix describes the special formats used.

## C. 1 Descriptor Lines

MAD writes the following descriptors on all tables:
COMMENT The current title string from the most recent TITLE command.
ORIGIN The version of MAD used.
DATE The date of the MAD run.
TIME The wall clock time of the MAD run.
TYPE The type of the table:
TWISS Lattice function table.
OPTICS Output of the OPTICS command.
TRACK Track table.
TUNES Table of tunes and chromaticities versus $\Delta E / p_{0} c$.
The track tables contain the following additional descriptors:
BETX Horizontal $\beta_{x}$,
ALFX Horizontal $\alpha_{x}$,
BETY $\quad$ Vertical $\beta_{y}$,
ALFY Vertical $\alpha_{y}$,
$\mathrm{X} \quad$ Horizontal orbit position,
PX Horizontal orbit slope,
Y Vertical orbit position,
PY Vertical orbit slope,
DX Horizontal dispersion,
DPX Horizontal dispersion slope,
DY Vertical dispersion,
DPY Vertical dispersion slope,
The format of descriptor lines is

$$
(, @,, A 16, ',, A 4, \ldots)
$$

Where the A16 format is used for the descriptor name, the A4 format for its format, and the remaining information uses the format listed below for columns.

## C. 2 Column Formats

The column formats used are listed in Table C.1. Control lines begin with the TFS control character,

Table C.1: Column Formats used in TFS Tables

| C format | Meaning | FORTRAN format |
| :---: | :---: | :---: |
| \%hd | Short integer | (I8) |
| $\% \mathrm{lf}$ | Long float | (G20.12) |
| \%f | Short float | (G14.6) |
| $\% \mathrm{ks}$ | String of length k | $\left(, \prime \prime, A, \prime^{\prime \prime}\right)$ |

followed by a blank. Data lines begin with two blanks. Columns are also separated by one blank character. The column width is chosen such as to accommodate the large of the column name and the data values of the column. Thus an integer column uses at least 8 characters to accommodate the (I8) format, but if its name has more than 8 characters, it becomes wider.

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[^0]:    ${ }^{1}$ Contributed by J. M. Veuillen (1987).

[^1]:    ${ }^{1}$ contributed by E. Nordmark (1985).

[^2]:    ${ }^{1}$ Adapted to MAD by D. Schofield (1983) and J.M. Veuillen (1988); rewritten extensively 1989.

